Important Summation Formulas

$$\sum_{i=l}^{u} 1 = 1 + 1 + ... + 1 = u - l + 1; \quad thus, \sum_{i=1}^{n} 1 = n$$

$$\sum_{i=1}^{n} i = 1 + 2 + ... + n = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n} i^2 = 1^2 + 2^2 + ... + n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{i=1}^{n} i^k = 1^k + 2^k + ... + n^k = \frac{1}{k+1} n^{k+1}$$

$$\sum_{i=0}^{n} a^i = 1 + a + ... + a^n = \frac{a^{n+1} - 1}{a - 1} (a \neq 1); \quad thus, \sum_{i=0}^{n} 2^i = 2^{n+1} - 1$$

$$\sum_{i=1}^{n} i 2^i = 1 * 2 + 2 * 2^2 + ... + n 2^n = (n-1) 2^{n+1} + 2$$

$$\sum_{i=1}^{n} \frac{1}{i} = 1 + \frac{1}{2} + ... + \frac{1}{n} \approx \ln n + \gamma, \text{ where } \gamma \approx 0.5772 \text{ (Euler's constant)}$$

$$\sum_{i=1}^{n} \log i \approx n \log n$$

Sum Manipulation Rules

$$\sum_{i=l}^{u} c a_i = c \sum_{i=l}^{u} a_i$$

$$\sum_{i=l}^{u} (a_i \pm b_i) = \sum_{i=l}^{u} a_i \pm \sum_{i=l}^{u} b_i$$

$$\sum_{i=l}^{u} a_i = \sum_{i=l}^{m} a_i + \sum_{i=m+1}^{u} a_i, \text{ where } l \leq m < u$$

$$\sum_{i=l}^{u} (a_i - a_{i-1}) = a_u - a_{l-1}$$