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October 1 Lecture

Announcements

- Midterm is on 10/10 (next Wednesday) in LECTURE.
- Midterm Review Session on Monday 10/8, 6-8pm, 2040 VLSB
- HW 3 due on Wed. Next HW assigned on 10/8 (Mon)

Wrap-up from Friday's Lecture:

g) a pair (aabcd)

1. pick ranks: 13 ways to choose what a should be

$\binom{12}{3}$ ways to choose b, c, d, the ranks which we have only 1 of

$13 \times \binom{12}{3} = 2860$

2. pick suits: all diff ranks, so 4 suit options for each.

$$\underbrace{\binom{4}{2}}_{a} \times 4 \times 4 \times 4 = 384$$

$= 2860 \times 384 = 1{,}098{,}240$

h) none of the above. could do $\binom{52}{5} - $ (all above)

or: all 5 cards must be distinct, but not consecutive. : $\binom{13}{5} - 10$

any suit combo is OK except all same: $4^5 - 4$

$$\left[\binom{13}{5} - 10\right]\left[4^5 - 4\right] = 1{,}302{,}540$$

Probability

Some background: most Stat 20/21 courses do not do this short bit on counting. However, what makes probability so hard for most students, I've found, is that they don't realize when they are over- or under-counting.

From this point on, we will loosely be following the book's treatment of probability. I will let you know when we start following the textbook more closely, and which chapters.

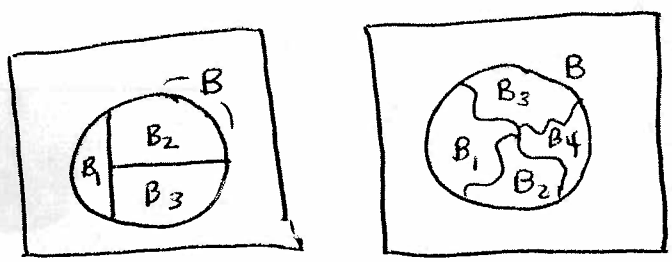
Defn. The probability of an event A is the number of outcomes corresponding to A, divided by the total # of outcomes.

The rules of probability

1. They are nonnegative. $P(B) \geq 0$
2. Addition. If $B_1, B_2, \ldots, B_n$ is a partition of B, then

$$P(B) = P(B_1) + P(B_2) + \ldots + P(B_n)$$

(B is partitioned into n events $B_1, B_2, \ldots, B_n$ if $B = B_1 \cup B_2 \cup \ldots \cup B_n$, and the events $B_1, B_2, \ldots, B_n$ are mutually exclusive.)



Examples of Partitions

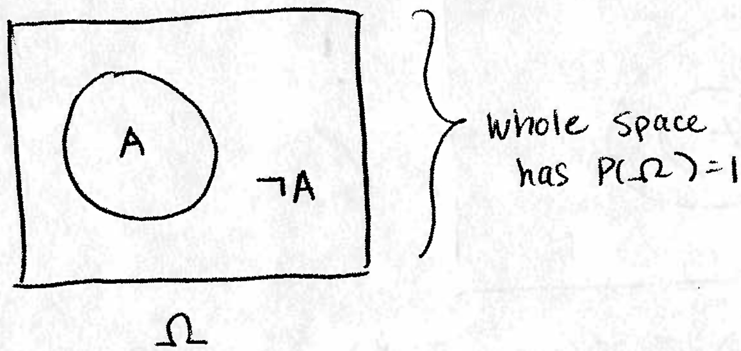
3. If Ω represents all possible outcomes, $P(\Omega) = 1$. (ie., all probabilities must be ≤ 1, and a partition of the outcomes must have probabilities that add to 1)

Example $\underbrace{P(H) + P(T) = 1}_{\text{single coin flip}}$ $\underbrace{P(\text{roll } 1,2,3) + P(\text{roll } 4) + P(\text{roll } 5,6) = 1}_{\text{single die roll}}$

Defn. A probability distribution (or distribution, for short) is an assignment of probabilities to outcomes s.t. the probabilities follow the rules above.

Frequently Used Techniques

Complement rule: $P(\text{not } A) = P(\neg A) = 1 - P(A)$



$P(\Omega) = 1 = P(A) + P(\neg A)$

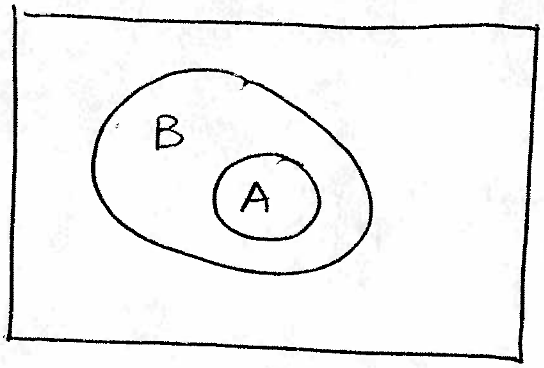
- $P(\text{rain}) = 1 - P(\text{not rain})$
- $P(\text{at least one H in 3 tosses}) = 1 - P(\text{no H in 3 tosses})$

The complement rule is frequently used in these cases:

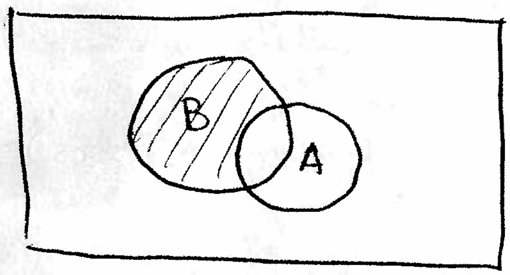
Event	Complement	Event	Complement
at most n ($\leq n$)	greater than n ($>n$)	no ___	at least one ___
at least n ($\geq n$)	less than n ($<n$)	all ___	not all ___ ($\neq$ none)

Be careful! The complement of "all H" or "no 6's" is NOT "no H" or "all 6's". You are thinking of the opposite, not the complement.

The difference rule:



$P(B \text{ and not } A) = P(B) - P(A \cap B) = P(B) - P(A)$
(in this case)

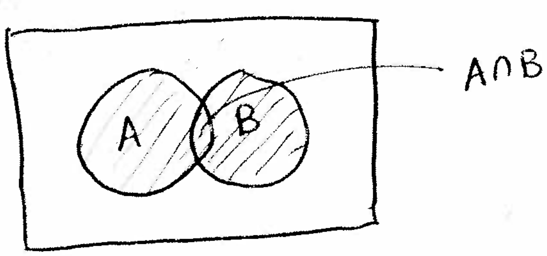


$P(B \text{ and not } A)$
$= P(B) - P(A \cap B)$
(general rule)

Ex. B = roll a # less than 6
A = roll an odd number
B and not A = roll <6 and (not an odd #)
= roll <6 and even #
$= \{2, 4\}$

Addition rule (Inclusion-Exclusion)

$P(A \text{ or } B) = P(A) + P(B) - P(A \cap B)$



Examples

Ex 1. In a certain population, 10% of the people are rich, 5% are famous, and 3% are rich and famous. For a person picked at random from this population:

(a) What is the chance that the person is not rich?

- P(not rich) = 1 − P(rich), since

P(rich) + P(not rich) = 1

$\Rightarrow$ P(not rich) = 1 − 0.10 = 0.90 or 90%

(b) What is the chance that the person is rich but not famous?

$$P(\text{rich but not famous}) = \underbrace{P(\text{rich})}_{0.10} - \underbrace{P(\text{rich and famous})}_{0.03} = 0.07 \text{ or } 7\%$$

In other words:

P(rich) = P(rich and famous) + P(rich and not famous)

(c) What is the chance that the person is either rich or famous?

P(rich or famous) = P(rich) + P(famous) − P(rich and famous)

= 0.10 + 0.05 − 0.03

= 0.12 or 12%

Named Distributions

There are some special distributions that appear in a wide variety of contexts and are given special names. We've already encountered one, the normal distribution. Here are a few others.

Bernoulli (p) Distribution For p between 0 and 1, this is the distribution on $\{0,1\}$ defined by the following distribution table:

outcome	0	1
probability	$1-p$	p

You can check this obeys the 3 rules of a probability distribution:

1. all of the probabilities are between 0 and 1.
2. There's only one possible way to partition the total set of outcomes: $\{0\}, \{1\}$.
3. $P(0) + P(1) = 1$
   (sum of the probabilities for all outcomes = 1)

We say that X has a Bernoulli(p) distribution if X has the outcomes above w/ probabilities as given above, ie.

$X = 1$ w/p p and $X = 0$ w/p $1-p$.

Also written as $X \sim$ Bernoulli(p).

Ex. Coin toss.

If X = # of H in a single coin toss.
$X \sim$ Bernoulli($1/2$) since $X = 1$ w/p $1/2$, and 0 w/p $1/2$.

If the coin is biased and comes up H $1/3$ of the time,
$X \sim$ Bernoulli($1/3$).

Uniform over a finite set w/ n outcomes

The probability of each outcome is $1/n$. If it's uniform over a set of #'s, like $\{1,2,3,\dots,n\}$, we write for shorthand $Unif\{1,2,\dots,n\}$.

Most common example: die roll.

A single die roll has a $Unif\{1,2,\dots,6\}$ distribution.

The probability of getting any one of the 6 faces is $1/6$.

We'll be talking about some more distributions, like the binomial/multinomial, the geometric, and hypergeometric (I hope!).

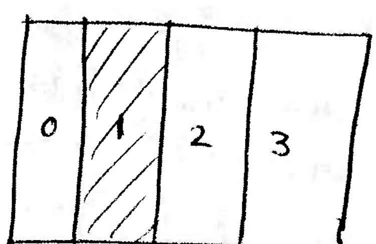
Some Examples

Ex. Complement rule:

P(exactly one 6 in 3 rolls) = 1 − P(0, 2, or 3 sixes)

The complement of this event is NOT (no 6's in 3 rolls).

If you did not get exactly one 6, you could have gotten 0 6's, 2 6's, or 3 6's.



all possibilities
Shaded: exactly one 6

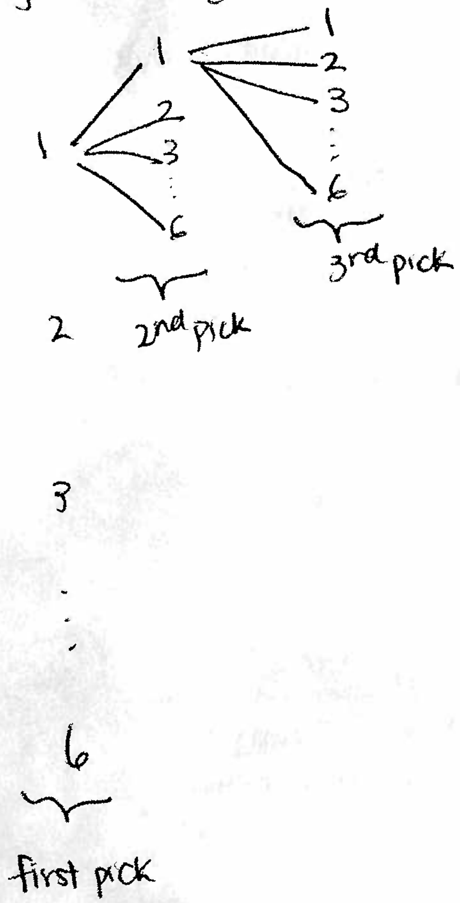
Ex. P(exactly one 6 in 3 rolls) is easier to find directly.

We need to find the # total outcomes and the # of ways to get exactly one 6 in 3 rolls. The way you count the denominator must match the top. More on this in the next example.

(8)

There are $6 \times 6 \times 6 = 6^3$ possible outcomes from 3 die rolls.

Counting this way is like making a tree:



Under this counting method, the sequence 1, 2, 3 is different from 2, 1, 3, because the former means your 1st pick was 1, then 2, whereas the latter means your 1st pick was 2, then 1. So, in the way we've counted, order matters.

How many outcomes correspond to "exactly one six in 3 rolls"? We'll need to count this taking order into account as well, because that's what we did for the denominator.

if we just wanted:

$6 \ \underline{\text{not } 6} \ \underline{\text{not } 6}$

that could occur in $1 \times 5 \times 5$ ways.

But there's $\binom{3}{1}$ places for the 6 to go, since we care about order. That gives us a total of $\binom{3}{1} \times 5^2$ ways,

and $P(\text{exactly one } 6 \text{ in } 3 \text{ rolls}) = \binom{3}{1} \frac{5^2}{6^3} = \binom{3}{1}\left(\frac{1}{6}\right)\left(\frac{5}{6}\right)^2$