

# Biconditionals and Logical Equivalence

## Definition: Logically Equivalent (2)

Propositions  $p$  and  $q$  are  $(p \equiv q)$  if  $p \leftrightarrow q$  is a tautology.

Example(s):  $p \equiv (p \wedge q) \vee p$

$p$	$q$	$p \wedge q$	$(p \wedge q) \vee p$	$p \leftrightarrow (p \wedge q) \vee p$
T	T	T	T	T
T	F	F	T	T
F	T	F	F	T
F	F	F	F	T

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## De Morgan's Laws

$$(1) \overline{p \wedge q} \equiv \bar{p} \vee \bar{q}$$

$$(2) \overline{p \vee q} \equiv \bar{p} \wedge \bar{q}$$

Example(s):

$$\overline{a \vee (b \vee c)} \equiv \bar{a} \wedge \overline{(b \vee c)} \text{ [De Morgan]}$$
$$\equiv \bar{a} \wedge \bar{b} \wedge \bar{c} \text{ [De Morgan]}$$

$$(p \vee q) \equiv r \equiv \bar{p} \vee (q \vee r)$$

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# Example: De Morgan's Laws and Programming

Checking to see if a score is not a 'B':

$$(x < 80) \vee (x \geq 90)$$

Alternatively

$$\neg(x \geq 80 \wedge x < 90)$$

$$\begin{aligned}
 p \vee q &\equiv \overline{\overline{p \vee q}} \quad [\text{Double Negation}] \\
 &\equiv \overline{\overline{p} \wedge \overline{q}} \quad [\text{De Morgan's}] \\
 &\equiv \overline{\overline{p}} \vee \overline{\overline{q}} \quad [\text{De Morgan's}] \equiv p \vee q \quad [\text{Double Neg.}]
 \end{aligned}$$



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## Common Logical Equivalences (1 / 3)

Table I: Some Equivalences using AND ( $\wedge$ ) and OR ( $\vee$ ):

(a)	$p \wedge p \equiv p, \quad p \vee p \equiv p$	Idempotent Laws
(b)	$p \vee \mathbf{T} \equiv \mathbf{T}, \quad p \wedge \mathbf{F} \equiv \mathbf{F}$	Domination Laws
(c)	$p \wedge \mathbf{T} \equiv p, \quad p \vee \mathbf{F} \equiv p$	Identity Laws
(d)	$p \wedge q \equiv q \wedge p$ $p \vee q \equiv q \vee p$	Commutative Laws
(e)	$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$ $(p \vee q) \vee r \equiv p \vee (q \vee r)$	Associative Laws
(f)	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$ $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$	Distributive Laws
(g)	$p \wedge (p \vee q) \equiv p$ $p \vee (p \wedge q) \equiv p$	Absorption Laws

Table II: Some More Equivalences (adding  $\neg$ ):

(a)	$\neg(\neg p) \equiv p$	Double Negation 
(b)	$p \vee \neg p \equiv \mathbf{T}, \quad p \wedge \neg p \equiv \mathbf{F}$	Negation Laws
(c)	$\neg(p \wedge q) \equiv \neg p \vee \neg q$ $\neg(p \vee q) \equiv \neg p \wedge \neg q$	De Morgan's Laws 

## Common Logical Equivalences (2 / 3)

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Table III: Still More Equivalences (adding  $\rightarrow$ ):

(a)	$p \rightarrow q \equiv \neg p \vee q$	Law of Implication
(b)	$p \rightarrow q \equiv \neg q \rightarrow \neg p$	Law of the Contrapositive
(c)	$p \equiv \mathbf{T} \rightarrow p$	"Law of the True Antecedent"
(d)	$\neg p \equiv p \rightarrow \mathbf{F}$	"Law of the False Consequent"
(e)	$p \rightarrow p \equiv \mathbf{T}$	Self-implication
(f)	$p \rightarrow q \equiv (p \wedge \neg q) \rightarrow \mathbf{F}$	Reductio Ad Absurdum
(g)	$\neg p \rightarrow q \equiv p \vee q$	
(h)	$\neg(p \rightarrow q) \equiv p \wedge \neg q$	
(i)	$\neg(p \rightarrow \neg q) \equiv p \wedge q$	
(j)	$(p \wedge q) \rightarrow r \equiv p \rightarrow (q \rightarrow r)$	Exportation Law
(k)	$(p \wedge q) \rightarrow r \equiv (p \rightarrow r) \vee (q \rightarrow r)$	
(l)	$(p \vee q) \rightarrow r \equiv (p \rightarrow r) \wedge (q \rightarrow r)$	
(m)	$p \rightarrow (q \wedge r) \equiv (p \rightarrow q) \wedge (p \rightarrow r)$	
(n)	$p \rightarrow (q \vee r) \equiv (p \rightarrow q) \vee (p \rightarrow r)$	

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## Common Logical Equivalences (3 / 3)

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Table IV: Yet More Equivalences (adding  $\oplus$  and  $\leftrightarrow$ ):

(a)	$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$	Definition of Biimplication
(b)	$p \oplus q \equiv (p \wedge \neg q) \vee (\neg p \wedge q)$	Definition of Exclusive Or
(c)	$p \leftrightarrow q \equiv \neg(p \oplus q)$	
(d)	$p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$	
(e)	$p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q$	
(f)	$\neg(p \leftrightarrow q) \equiv p \leftrightarrow \neg q \equiv \neg p \leftrightarrow q$	

Remember: You **do not** need to memorize these three tables ...

... But you **do** need to know how to use them!

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# Applications of Logical Equivalences (1 / 6)

**Question: Is  $(p \wedge q) \rightarrow p$  a tautology? (1)**

By use of a Truth Table; we've seen this before:

$p$	$q$	$p \wedge q$	$p$	$(p \wedge q) \rightarrow p$
T	T	T	T	T
T	F	F	T	T
F	T	F	F	T
F	F	F	F	T

Because the expression evaluates to true for all possible arrangements of truth values, the expression is a tautology.

# Applications of Logical Equivalences (2 / 6)

**Question: Is  $(p \wedge q) \rightarrow p$  a tautology? (2)**

$$(p \wedge q) \rightarrow p \equiv (p \rightarrow p) \vee (q \rightarrow p) \quad [\text{TIILK}]$$

$$\equiv T \vee (q \rightarrow p) \quad [\text{self Imp.}]$$

$$p \vee T \equiv T \quad [\text{Domination}]$$

[negation law]

[Domination]

# Applications of Logical Equivalences (1/6)

Question: Is  $(p \vee q) \rightarrow p$  a tautology? (1)

By use of a truth table, we've seen this before:

$p$	$q$	$p \vee q$	$(p \vee q) \rightarrow p$
T	T	T	T
T	F	T	T
F	T	T	F
F	F	F	T

$$(p \wedge q) \rightarrow p \equiv p \rightarrow (q \rightarrow p)$$

[T III L j]

$$\equiv p \rightarrow (\neg q \vee p)$$

[Law of Implication]

$$\equiv \neg p \vee (\neg q \vee p)$$

[Law of Implication]

$$(q \leftarrow p) \vee (q \leftarrow q) \equiv (\neg p \vee p) \vee \neg q$$

$$(q \leftarrow p) \vee T \equiv [Comm. and Assoc.]$$

$$T \equiv T \vee \neg q$$

[negation law]

$$\equiv T \quad [Domination]$$

## Applications of Logical Equivalences (3 / 6)

Question: Is  $(p \wedge q) \rightarrow p$  a tautology? (3)

By reasoning:

When  $p$  is true:

$$(T \wedge q) \rightarrow T \quad \text{Because anything} \rightarrow T \text{ is } T.$$

When  $p$  is false:

$$(F \wedge q) \rightarrow F \equiv F \rightarrow F$$

Because  $F \rightarrow F \equiv T$

9/5/12  
↑

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## Applications of Logical Equivalences (4 / 6)

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9/2/12

Example(s):

if (games <= 10 || ties > 2) && games >= 11) ...

$p$ :  $games \leq 10$

$q$ :  $ties > 2$

if  $(p \vee q) \wedge \neg p$

$$\neg (p \wedge \neg p) \vee (q \wedge \neg p) \quad [\text{dist.}]$$

$$\neg F \vee (q \wedge \neg p) \quad [\text{negation}]$$

$$(q \wedge \neg p) \quad [\text{Identify}]$$

if ( $ties > 2$  and  $games \geq 11$ ) ...

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