Physics 2213: Electromagnetism, Fall 2012

Problem Set # 1

(Due Friday, August 31 at 5:00pm *sharp.*)

Agenda and readings

Readings marked YF refer to sections from the text book, *University Physics*, 13th edition, volume 2, by Young and Freedman.

- Lec 1 8/23: Electric Charge and Force.
  Reading: YF 21.1-3
- Lec 2 8/28: Coulomb's Law, Superposition, Electric field concept
  Reading: YF 21.4-5
- Lec 3 8/30: Computing Electric Fields, Electric Field Lines
  Reading: YF 21.5-7 (21.7 very useful for Problem Set #2)

Notes: This assignment is shorter than it appears. Much of it provides background material, coaching and review of key points from lecture so that the assignment is relatively self-contained. To help identify what the assignment asks of you, things you should do are marked with underlines, *hints* appear in *italics*, and **important information** appears in **boldface**.

Contents

1 Getting a feel for magnitudes and units

This problem will give you some basic sense for the magnitudes of basic quantities involving charge and the flow of charge. This problem also is designed to give you some preview of concepts that we will soon learn about in more detail. This way, you'll already have some idea of what we are talking about when we learn about these things in depth.

1.1 Coulombs and amperes

The coulomb (C) is defined as the amount of charge that flows through a circuit with a current (flow of charge per unit time) of one ampere (A) in a time period of one second (s), so that 1 A=1 C/s. The modern definition of the ampere involves magnetic fields and is beyond our present discussions. (We will come back to this definition later.) The older definition (still the legal definition in the US) was that one ampere of current would deposit 1.118 mg ($= 1.118 \times 10^{-3}$ g$= 1.118 \times 10^{-6}$ kg) of silver per second when electroplating silver from a solution of silver nitrate solution, where each deposited silver atom is associated with the flow of a single electron (basically because silver is in the form of singly charged ions, Ag^+, each of which has one fundamental unit of charge).

The bottom line of all of this, is that 1.118 mg of Ag^+ ions has a charge that is quite close to $q = +1$ C. Using Avogadro's number and the atomic mass of silver of 107.8682 g/mole, determine (a) how many fundamental units of charge make up 1 coulomb. From this value, (b) determine the fundamental unit of charge e in units of Coulombs.

Lessons: The presence of Avogadro's number in your calculation should help you understand why the fundamental unit of charge is so small in Coulomb units. Also, it should help you see why a measurement of the fundamental unit of charge actually led to the first accurate measure of Avogadro's number.

1.2 How big is an ampere in engineering applications?

Electric vehicles are one alternate technology currently being pushed by the US government to reduce dependence on foreign oil. (You can get some basic engineering numbers and learn about electric vehicle conversion kits at www.ddmotorsystems.com/ElectricVehicles.php . You can also see a world-record holding converted Porsche 944 at http://www.jstraubel.com/944EV/EVproject.htm).

The motors on these vehicles (well, maybe not the Porsche but commercial vehicles) typically put out about 25 kilowatts (about 35 horse power — which may not sound like a lot, but electric motors are much more efficient at lower speeds) and operate around 100 volts. (For reference, the voltage that comes out of your wall sockets is close to this, around 120 V.)

Now, you will recall from your mechanics class that 1 watt $\equiv$ 1 W = 1 J/s is a measure of *power*, energy (measured in joules) per unit time (measured in seconds). The volt is a new unit. As we will study in depth later, the volt is a measure of the energy (measured in joules) associated with moving one unit of charge (measured in coulombs), 1 volt $\equiv$ 1 V = 1 J/C.

1.2.1 Current and cruising range in an electric vehicle

As you might expect, the peak operating current in an electric vehicle is quite large as far as typical electric circuits go. From the typical power (energy/time) and voltage (energy/charge) in an electric vehicle as given above, (a) determine the typical current (charge/time flowing through the motor). A typical lead-acid car battery is rated at 120 amp-hours of storage. For how long a time (b) could a car like this travel with a storage of only 120 amp-hours? (c) At typical highway speeds, about how far could such a car go on such a battery pack?

Lessons: This problem should give you a sense for why new battery technologies are key to making electric vehicles practical. (Note that it would take eight (8) regular car batteries to make such a 100 V battery pack!) Also, this should give you an idea why the amperes are a good unit for measuring current in motor applications. (Electronic applications, on the other hand, actually use far less current, more like a milliampere.)

1.3 Size of the coulomb

As the left side of Figure 1 shows, under operating conditions in an electric vehicle, positive charge flows out of the positive terminal of the battery, travels through the electric motor *and then returns* back to the battery at its negative terminal. Charge therefore travels around and around this way without any significant build up of charge. As we are about to see, having a such a "complete circuit" is key to having any significant flow of charge.

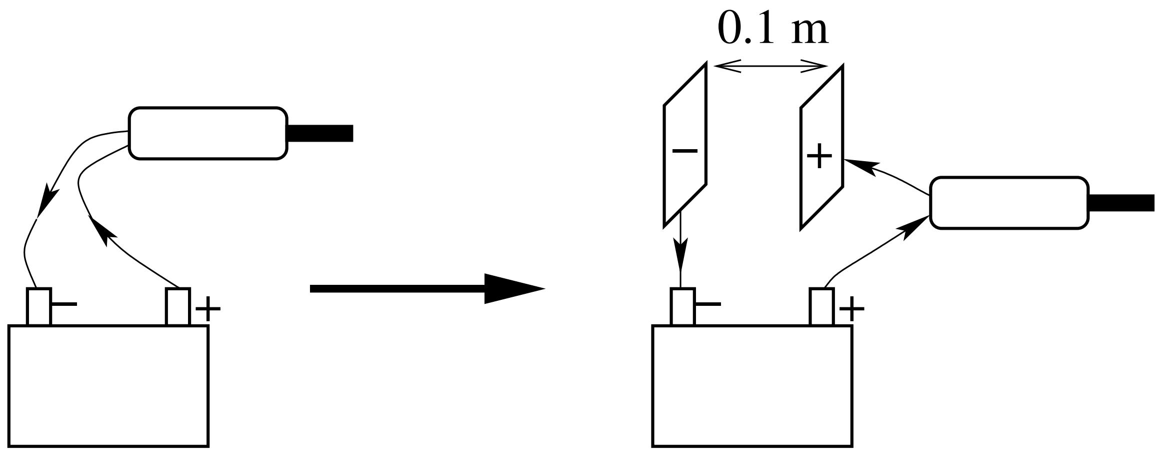

Figure 1: Flow of charge in an electric car (left), imagined version where charge is not allowed to flow back to the battery (right).

Suppose for a moment that, instead of a complete circuit, the circuit is broken. The battery will then still pull charge into its negative terminal from the body of the car (to which is it always connected, or "grounded"), leaving the body negatively charged, as shown schematically on the right side of the figure as a plate with a negative charge. Charge will then flow through the motor (as shown) but then build up as a net positive charge at the end of the "broken" wire. (Shown again schematically as a plate with a positive charge.) Just for the purposes of making a *rough estimate*, lets suppose that the negative and positive charges act like point charges separated by a typical distance in a car of r=0.1 meter. Finally, suppose that the same current as you found in Problem 1.2.1 flows for just 1 ms (0.001 seconds).

Under the above conditions, how much charge Q would have flowed from the negative plate through the battery and onto the positive plate? What would be (b) the Coulomb force be between the resulting built up positive and negative charges? Then, (c) determine the mass, in metric tonnes (1 mt $\equiv$ 1,000 kg), whose weight would correspond to this force. (Remember that 1 metric tonne of mass has a weight of about 2,200 lbs in English units, which is about one ordinary English ton.) Finally, (d) do you think the car frame would be able to sustain such a force?

Lesson: This should convince you that 1 coulomb is an extremely large amount of charge and that only "complete" circuits (where flowing charges return to the battery) are able to move such large amounts of charge because they do not require charge to actually build up.

1.4 More realistic charge build up in the car

Of course, when you have a circuit with an open connection as on the right side of Figure 1, you will end up with small amounts of charge build up. Indeed, in any device with an "open switch" (switch in the "off" position), or even at the terminals of a disconnect battery, there will also be small amounts of charge build up. (In fact, even in a complete circuit, some charge will build up at the various points around the circuit.)

To get an idea of how much charge this might realistically involve, suppose the charge on each of the plates in the figure was Q=0.1 nC (0.1×10^{-9} C) and answer the following questions. What would be (a) the force between the plates be then? (Again, modeling them as point charges.) Now, use the formula

$$\text{energy} = \text{force} \times \text{distance},$$

to (b) estimate how much energy is needed to separate this much charge by the distance of 0.1 m. Next, (c) how many volts (energy per unit charge moved) is this? And, finally, (d) could the 100 V battery pack in an electric vehicle reasonably provide this amount of energy to the charges it moved?

Now, (e) repeat the above analysis using variables (k_C, Q and r) instead of numbers to estimate the voltage V needed to move the net amount of charge Q, and (f) solve your equation to find a numerical value for how much charge Q a 100 V battery could move under these circumstances.

Finally, under these more realistic charge conditions, (g) how much force would there be between the plates? And, (h) about how long (at the amperage you estimated above) in seconds would it take for this build up to occur?

Lessons: The above exercise should convince you that such "stray charges" like the above build up very quickly but do not result in noticeable forces.

2 Millikan's Nobel Prize Experiment

Problem 1.1 showed you how to determine the fundamental unit of charge, if you had accurate knowledge of Avogadro's number. Historically, things went the other way: it is quite hard to count how many atoms make up a gram of something, but you can actually make a quite accurate and direct measurement of the charge on the electron with a fairly primitive apparatus, as Robert Millikan did in 1913 to earn him the Nobel prize. (You can read about this on Wikipedia at en.wikipedia.org/wiki/Millikan_Experiment .)

Through his famous experiment, Millikan also was the first to observe directly that all observable charges come as a multiple of a fundamental unit of charge (namely that of an electron). Knowing this fundamental unit then allowed him to give the first accurate value for Avogadro's number and another important fundamental constant, the mass of the electron. (The electron charge-to-mass ratio was already known through experiments we'll learn about later.) These are all very important numbers and have provided the basis for many technological advances in the years since. In this problem, you will recreate Millikan's analysis using (as is necessary at this early stage in the course) a highly idealized apparatus. We'll come back and re-investigate this real apparatus later in the course.

Figure 2 shows the idealized experimental situation we shall consider. The apparatus consists of a fixed positive charge of value $Q = +10$ nC $\equiv 10 \times 10^{-9}$ C. (The actual experiment used a planar electrode, which we'll learn about later in the course.). It is found that tiny drops from a fine mist of oil (which pick up different charges by rubbing against the nozzle creating the mist) of different masses m will "hover" in the air, apparently weightless, at a distance of d =5 mm from the charge Q. Table 1 gives the masses m (measured in units of ng$\equiv 10^{-9}$ g=10^{-12} kg) of twenty such drops. (How the masses are measured in the actual experiment is a long story you can read on Wikipedia!)

0.31256	0.62422	0.23492	0.46763	0.23339
0.23476	0.54728	0.54642	0.62455	0.31437
0.15561	0.70245	0.31187	0.70182	0.31314
0.15521	0.46762	0.54825	0.54579	0.15757

Table 1: Masses of hovering droplets in Millikan charge experiment

Based on this data, what is the value of the fundamental unit of charge e?

Note: The value you get will be of the same order of the actual electron charge **but** we've changed the data

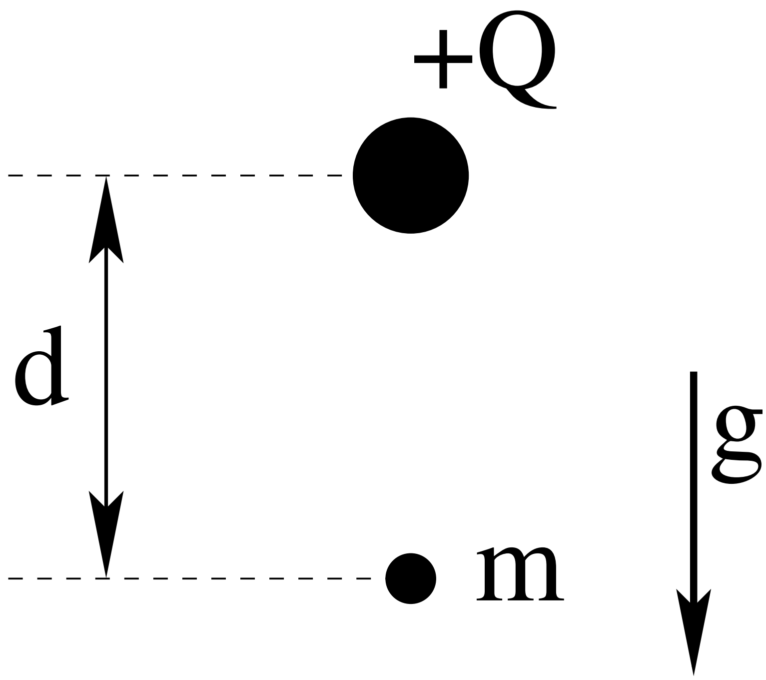

Figure 2: Idealized Millikan oil drop experiment

in this problem a bit so that you won't get the actual value for a real electron!!!
Hints: If you make a graph of all of the values that you get for the charges on the oil drops (sorted in order of their size) you should be able to see a pattern that lets you figure out the fundamental charge. Be careful, though, because, just like in an actual experiment, we've added small random errors to the masses.
Contest: If you use all of the data to derive your final answer (rather than just a single value or two) you can get a very accurate number. Go to post 39 (https://piazza.com/class#fall2012/phys2213fall/39) on Piazza and fill out the form to see who gets the closest in your section and in the entire class!

3 Polarization

In this problem you will explore some common polarization phenomena, both qualitatively and semi-*quantitatively*. (Later in the semester, we will be able to say a lot more about these phenomena once we learn additional concepts such as the electric field, electric field lines, and potential.)

Figure 3 shows what happens when a charge $+Q$ is placed at a distance R from the center of a **solid** (i.e., not hollow) conducting metal sphere of radius r that has no net charge on it. Because charges are free to move in the conductor, any negative charges in the conductor which are free to move are attracted to the external charge $+Q$ and collect on the side nearest to it. Because there is no net charge on the sphere, this implies that there must be some region of net positive charge (the region from which the negative charges have moved away) on the sphere as well. Because such a region would be repelled by the positive external charge $+Q$, it naturally occurs on the opposite side of the sphere, as in the figure. (In reality, the positive and negative charges will be distributed in a pattern all across the surface of the sphere. For the purposes of making a basic estimate, we'll assume for now that the negative and positive charges on the sphere all build up on the two points of the sphere nearest and furthest from the external charge.) Finally, we shall denote as q the magnitude of net amount of polarization charge which moves from the left to the right of the sphere.

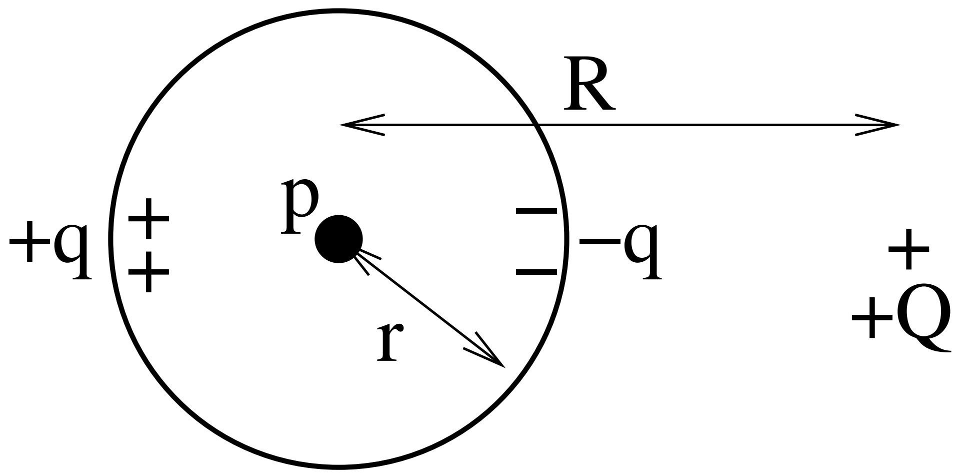

Figure 3: Model for polarization response of a solid spherical conductor near a positive external charge $+Q$

3.1 Net direction of force (qualitative)

Given the arrangement of charges shown in Figure 3, (a) in what direction will be the net force on the conducting sphere due to the presence of the charge $+Q$? Also, (b) in what direction will be the force on the charge $+Q$ due to the presence of the sphere? Explain your reasoning for both answers in a brief sentence or two.

3.2 Net direction of force (quantitative)

Derive an expression for the net force on the sphere in terms of no quantities other than Coulomb's constant k_C, the radius of the sphere r, the distance of the external charge from the sphere center R, the external charge Q, and the polarization charge q. Show mathematically that your expression for the force indeed has the sign that corresponds to your qualitative answer in Problem 3.1, given that both $q > 0$ and $Q > 0$.

3.3 Extent of polarization (qualitative)

What do you think would happen (answer "increase" or "decrease") to the magnitude of the polarization charge q that moves across the sphere if (a) you increase the distance R, (b) if you increase the charge Q, (c) you increase the radius R? Again, explain your reasoning for each answers with a brief phrase "because ...".

3.4 Extent of polarization (semi-quantitative)

We can actually make a pretty reasonable estimate of the amount of charge q that flows across the sphere through the following argument. Imagine that there is a charge carrier (one of the charges that is free to move in the conductor) of charge p in the center of the sphere. If there is any net force on that charge carrier, it will move either to the left or to the right and thereby increase or decrease q. This process will continue until there are no net forces left on any of the charge carriers in the sphere, only then will the motion of the carriers settle down into the situation known as "equilibrium". Once equilibrium is reached, we known, then, that the net force on any charge p in the center of the conductor must be zero. This then gives us a way to determine an estimate for q!

(a) Derive an expression for the total force on the charge p for the arrangement of charges show in in Figure 3 in terms of only p q, Q, r and R. Then, (b) set your expression for the total force to zero, and solve for the polarization charge q.
Hint: If you'd like to double check your formula for q, you should verify that it has all of the qualitative behaviors you described in Problem 3.3.
Note: This is a good working estimate, but only an estimate, because the actual positive and negative charges distribute themselves around the surface of the sphere and don't actually collect at just two opposite points.

3.5 Conducting sphere with a net charge

A favorite "trick question" for courses such as this is to ask about the direction of the force on the sphere when it has a net positive charge $+P$ on it. Depending on the amount of the charge $+P$ on the sphere, you can either get attraction or repulsion, and so you really cannot give the direction of the force without knowing more about the situation.

A reasonable assumption for making a first estimate of what will happen would be to imagine that the positive charge on the sphere ends up on opposite sides of the sphere (since these charges repel each other), with $+P/2$ on the far left and $+P/2$ on the far right point. (Actually, you get an even spread of these charges, with $+P/2$ distributed on the left and right halves of the sphere, respectively, so this is not too far from the truth.) In sum, for the purposes of this problem, we will assume that, after polarization, you'll have a charge $+P/2+q$ on the far left point and $+P/2-q$ on the far right point.

(a) Using the same arguments as in Problem 3.4 with the imaginary charge p at the center of the sphere, determine the amount of polarization q under these new conditions.
Hint: You'll find a familiar answer.

(b) **EXTRA-CREDIT CHALLENGE:** Now, compute the new net force on the sphere (remembering that the charges on the sides are now $+P/2 \pm q$) in terms of no variables other than P, Q, r, R and k_C, and solve for the value P where the net force goes from attractive to repulsive.
Hints: You'll get some pretty pretty messy expressions while you work though this, but don't give up, you can manage the algebra if you get everything on a common denominator. You'll find that the repulsive answer for your answer for the force goes like QP while the attractive part goes like Q^2. Next, you'll find that your answer for the cross-over value for P will equal Q times a function that depends only on the ratio r/R!

4 Introduction to electric field concept

In the second lecture, we will introduce the concept of the electric field. In short, the idea is that because the net electric force $\vec{F}_Q$ on a charge Q at location $\vec{r}$ is proportional to the charge, it makes sense to define a quantity $\vec{E}(\vec{r})$ which is the force per unit charge for a charge situated at point $\vec{r}$. The standard SI units for the electric field are thus newton/meter $\equiv$ N/m, and to find the force on the charge Q, you may simply use the formula $\vec{F}_Q = Q\vec{E}(\vec{r})$.

4.1 Warm-up with vectors (no field yet)

Do textbook problem 21.37 from *University Physics*, 13th edition.

4.2 Working from definition of electric field

Do textbook problem 21.25 from *University Physics*, 13th edition.

4.3 Working the definition backwards

Do textbook problem 21.28 from *University Physics*, 13th edition.

4.4 Electric field with vectors

Do textbook problem 21.30 from *University Physics*, 13th edition.