Cornell University Department of Physics

Physics 2213: Electromagnetism, Fall 2012

Problem Set # 8

(Due Friday, October 26 at 5:00pm *sharp.*)

Agenda and readings

IMPORTANT– The topics of electric field energy density and dielectric materials will not be covered in lecture, so it is **very important** that you read and study the following sections of the text book **very carefully**: Energy density in electric field YF 24.3, Dielectrics YF 24.4. This material is the subject of problems 1 and 2 on this problem set. In addition to the material in the text book it is covered in special lecture notes that are posted on Piazza as an attachment to the announcement entitled "Lecture Catch-up plan" @381.
Readings marked YF refer to sections from the text book, *University Physics*, 13th edition, volume 2, by Young and Freedman.

- Relativity and magnetic field, YF 27.1-2
- Force on a moving charges and currents in wires, YF 27.4-6
- Force on a line segment, and current loop: YF 27.7
- Biot-Savart Law: YF 28.1-2

Contents

1 Energy density

Do Exercise 24.34.

Add the following parts:

(d) Convert to variable names: Calling the potential difference V, and the inner and outer radii r_1 and r_2, respectively, find an expression for the capacitance.

(e) What is the energy density u at any distance r from the center?

(f) Integrate the energy density $U = \iiint u\, dV$ to find the total energy stored in the electric field within the capacitor.

(g) Verify that your answer in (f) obeys the standard capacitor result $U = \frac{1}{2}CV^2$

2 Dielectrics

(a) Do Exercise 24.38, YF.

(b) Do Exercise 24.35, YF.

(c) Do Exercise 24.37, YF.

3 Magnetic force on moving charges

3.1 Magnetic force on an electron

An electron moves with speed 6.0×10^5 m/s in a uniform magnetic field of strength 0.60 T that points east. At some instant, the electron experiences an upward magnetic force of magnitude 4.5×10^{-14}.

(a) In what direction is the electron moving at that instant? [If there is more than one possible direction, describe all the possibilities.] On a carefully drawn and labeled diagram, show the (possible) direction(s) of the electrons velocity relative to the magnetic field and the force directions. (Suggestion: Draw your diagram so that the force is perpendicular to the plane of the paper.)

(b)What is the greatest force that this magnetic field could possibly produce on an electron moving at this speed? Show the orientation of that electron's velocity relative to the field.

3.2 Magnetic force on a cable

A 30 m long, 0.50 cm diameter copper cable running east-west carries an electric current of 100 A directed east. At the location of the cable, the Earths magnetic field is directed northward and down at an angle of 45° below the horizontal.

(a) What are the magnitude and direction of the magnetic force that acts on the cable due to the Earth's magnetic field? Illustrate your answer with a carefully drawn and labeled diagram showing the cable, magnetic field, and magnetic force on a N-S, up-down set of coordinate axes as seen from the west end of the cable. Would other vantage points serve as well as this one for visualizing whats going on here?

(b) How does this force compare with the cables weight? Should this force be considered when designing mechanical supports for the cable? [Copper's density is 8.96×10^3 kg/m^3.]

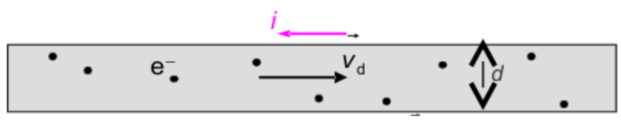
3.3 Magnetic force on a moving charge using unit vector notation

Do Exercise 27.8.

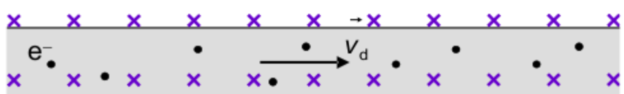
4 Magnetic force on moving charges

4.1 The Hall effect

The cartoon below shows a section of a metallic conducting strip of width d carrying a current i due to mobile electrons. On average the mobile electrons move with a drift velocity $\vec{v}_d$.



Now suppose that we turn on a uniform magnetic field $\vec{B}$ directed into ($\times$) the page as shown in the cartoon below.



(a) Just after the magnetic field is turned on, the mobile electrons, on average, will each experience a magnetic force. Please indicate and label the direction of this force on the cartoon above. On average, what is the magnitude of the magnetic force on a mobile electron?

The average magnetic force on each mobile electron causes it to move to one side of the conductor. Since the electrons cannot escape the conductor, an accumulation of negative charge builds up on this side of the conductor and, since the conductor as a whole is electrically neutral, a positive charge of equal magnitude is left on the opposite side of the conductor. This separation of positive and negative charge creates an electric field component across the conductor known as the Hall field $\vec{E}_{\mathrm{H}}$ (named after its discoverer, American physicist Edwin Hall). (Note that $\vec{E}_{\mathrm{H}}$ is not the total electric field in the conductor. There is also an electric field component directed along the length of the conductor due to a power supply or battery that drives the current in the conductor.)

(b) On the cartoon directly above Part (a), indicate and label the direction of $\vec{E}_{\mathrm{H}}$. What is the direction of the electric force on a mobile electron due to $\vec{E}_{\mathrm{H}}$? Indicate and label this electric force direction on the cartoon directly above Part (a). What is the magnitude of the electric force on a mobile electron due to $\vec{E}_{\mathrm{H}}$?

(c) As more and more charge separation accumulates across the conductor the Hall field component grows stronger and stronger until, on average, the electric force on a mobile electron due to $\vec{E}_{\mathrm{H}}$ balances the magnetic force on the electron. At this point, a steady state is reached in which mobile electrons are no longer deflected to one side of the conductor and the Hall field stops growing. What is the drift speed $\vec{v}_d$ of the mobile electrons in terms of B and the steady-state magnitude of the Hall field, $\vec{E}_{\mathrm{H,ss}}$? Hint: What should the Lorentz force on a mobile electron equal at steady state?

(d) The Hall field causes a potential difference V_{H} across the conductor that is known as the Hall voltage, $V_{\mathrm{H}} = \vec{E}_{\mathrm{H,ss}} d$, that can be measured with a voltmeter. Indicate on the cartoon directly above Part (a) where voltmeter leads should be connected in order to measure the Hall voltage.

(e) Let l be the thickness of the conductor (in the cartoons it's the dimension perpendicular to the plane of the page). If the value of B is known, then by measuring V_{H}, l, and the current i, the number density n (number per unit volume) of mobile electrons in the conductor can be calculated. Write an expression for n in terms of these quantities. Hint: Recall that the current density $\vec{J} = ne\vec{v}_d$. (Note that if the number density n for a given material is known then measurements of V_{H}, l, and i can be used to calculate B. This is the idea behind a Hall probe that you will use in lab to measure magnetic field.)

(f) Suppose that the conductor is made of some material in which the mobile charges are positively charged. For the same current and magnetic field directions shown in the cartoons, would these mobile positive charges move to the same or the opposite side of the conductor as did the mobile electrons in the original problem? Would the polarity of the Hall voltage be the same or reversed?

4.2 Electromagnetic flow meter

Blood flow rates can be measured during surgery using an electromagnetic flow meter. Blood containing ions (primarily Na^+ and Cl^-) flows through an artery with a diameter of 0.50 cm. The artery is oriented east-west in a magnetic field of 0.20 T pointing north and develops a Hall voltage of 0.32 mV across its diameter.

(a) Where should the two leads from the voltmeter be placed on the artery to measure the full Hall voltage? On a diagram with east-west and up-down axes, show how to make these electrical connections.

(b) If the blood is flowing east, which lead is at the higher potential?

(c) What is the blood speed (in m/s) and the volume flow rate (in m^3/s)?