Physics 211

Sections 1 & 70

Dr. Geoffrey Lovelace

Fall 2012

Lecture 24 (12/04/12)

Lecture 24 outline

- Announcements
- Angular momentum
  - Definition
  - Conservation
  - Examples & demos
- Bonus examples
  - Rotational energy / power / work
  - Torque
  - Rotational dynamics

Announcements

- Homework #10: due **today** at 11:59PM
- Homework #11: due December 13 at 11:59PM
- Reading: Start chapter 13: vibrations & waves
  - Last new chapter
- Office hours: 4PM-5PM today
  - **Bonus office hours today only**: 10AM-11AM
  - McCarthy Hall room 601B
- Final exam December 20, 9:30AM-11:20AM
  - Skip the final? See me in office hours!
  - Emphasize material since Exam #3 (cumulative)

Today

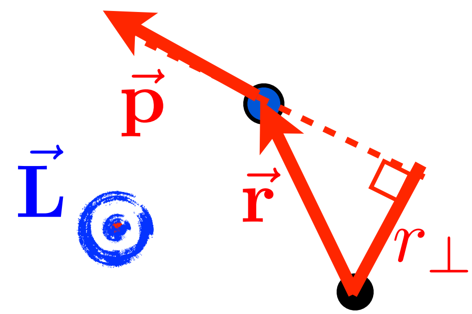
Date	Event
Nov 15	**Exam 3**
Nov 20	*Fall Recess — No class*
Nov 22	*Fall Recess — No class*
Nov 27	Rigid body rotation, torque
Nov 29	Rotational dynamics, rotational energy
Dec 4	Angular momentum, rigid body wrap-up *HW #10 due*
Dec 6	Harmonic motion
Dec 11	Harmonic motion & waves
Dec 13	Gravitational waves, harmonic motion, black holes, *HW #11 due*
Dec 20	**Final exam 9:30AM–11:20AM**

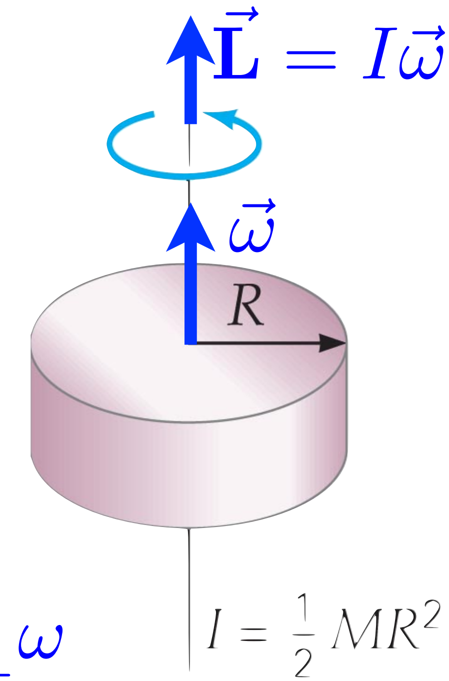
Lecture 24 outline

- Announcements
- Angular momentum
  - Definition
  - Conservation
  - Examples & demos
- Bonus examples
  - Rotational energy / power / work
  - Torque
  - Rotational dynamics

Angular momentum

- Units: $\mathrm{kg \cdot m^2/s}$
- Magnitude
  - Particle





$$L = r_\perp p = r_\perp mv = r_\perp m(r_\perp)\omega = mr_\perp^2\omega$$

  - Rigid body

$$L = \sum_i \left(m_i r_{\perp i}^2\right)\omega = I\omega$$

- Direction: right-hand rule, same as $\vec{\omega}$ so $\vec{\mathbf{L}} = I\vec{\omega}$

$\vec{\mathbf{p}}$ = linear momentum
$\vec{\mathbf{r}}$ = radial vector
$r_\perp$ = moment arm
$\vec{\mathbf{L}}$ = angular momentum
I = moment of inertia
$\vec{\omega}$ = angular velocity

Angular momentum

- Newton's second law & momentum

Linear momentum $\vec{\mathbf{p}}$

$$\vec{\mathbf{p}} = m\vec{\mathbf{v}}$$

$$\vec{\mathbf{F}}_{\text{net}} = \frac{\Delta\vec{\mathbf{p}}}{\Delta t}$$

Angular momentum $\vec{\mathbf{L}}$

$$\vec{\mathbf{L}} = I\vec{\omega}$$

$$\vec{\tau}_{\text{net}} = \frac{\Delta\vec{\mathbf{L}}}{\Delta t}$$

- Conservation laws

$$\vec{\mathbf{F}}_{\text{net}} = 0 \Rightarrow \vec{\mathbf{p}} = \text{const}$$

$$\vec{\mathbf{F}}_{\text{net}} = \text{net force}$$

$$\vec{\tau}_{\text{net}} = 0 \Rightarrow \vec{\mathbf{L}} = \text{const}$$

$$\vec{\tau}_{\text{net}} = \text{net torque}$$

  - No external net force: momentum conserved
  - No external net torque: angular momentum conserved

Clicker question #110

- A person sits on a slowly spinning stool holding weights, arms extended. What happens to the person's **angular momentum** after pulling the weights inward?

A Increases in magnitude

B Decreases in magnitude

C Changes direction

D Remains constant

Clicker question #111

- A person sits on a slowly spinning stool holding weights, arms extended. What happens to the person's **moment of inertia** after pulling the weights inward?

A Increases

B Decreases

C Depends on speed of weights

D Remains constant

Clicker question #112

- A person sits on a slowly spinning stool holding weights, arms extended. What happens to the person's **angular velocity** after pulling the weights inward?

A Increases in magnitude

B Decreases in magnitude

C Changes direction

D Remains constant

Angular momentum examples

- Figure skater
  http://www.youtube.com/watch?v=AQLtcEAG9v0

(b)

$$L \approx 25 \text{ kg} \cdot \text{m}^2/\text{s}$$

$$I \approx 6 \text{ kg} \cdot \text{m}^2 \qquad I \approx 2 \text{ kg} \cdot \text{m}^2$$

Example: skater

- If the skater's initial angular velocity is 40 rpm, what is it after reducing her moment of inertia as shown?

(b)

$I \approx 6\ \mathrm{kg} \cdot \mathrm{m}^2$ $I \approx 2\ \mathrm{kg} \cdot \mathrm{m}^2$

$$\vec{\mathbf{L}} = \text{angular momentum}$$
$$I = \text{moment of inertia}$$
$$\vec{\omega} = \text{angular velocity}$$

Given:

$$\omega_0 = 40\text{rev}/\text{min}$$
$$I_0 = 6\ \mathrm{kg} \cdot \mathrm{m}^2$$
$$I_f = 2\ \mathrm{kg} \cdot \mathrm{m}^2/\mathrm{s}$$

Goal: ω_f

Principle & eqns.

Conservation of angular momentum

$L = I\omega$ $L_f = L_0$

Example: skater

Given:

$$\omega_0 = 40\text{rev}/\text{min}$$
$$I_0 = 6.0\ \text{kg}\cdot\text{m}^2$$
$$I_f = 2.0\ \text{kg}\cdot\text{m}^2/\text{s}$$

Goal: ω_f

Principle & eqns.

Conservation of angular momentum

$$L = I\omega \qquad L_f = L_0$$

$$\vec{\mathbf{L}} = \text{angular momentum}$$
$$I = \text{moment of inertia}$$
$$\vec{\omega} = \text{angular velocity}$$

$$L_0 = I_0\omega_0 = I_f\omega_f$$

$$\omega_f = \frac{I_0}{I_f}\omega_f$$

$$\omega_f = \frac{6.0\ \text{kg}\cdot\text{m}^2}{2.0\ \text{kg}\cdot\text{m}^2}(40\ \text{rev}/\text{min})$$

$$= 120\ \text{rev}/\text{min}$$

Clicker question #113

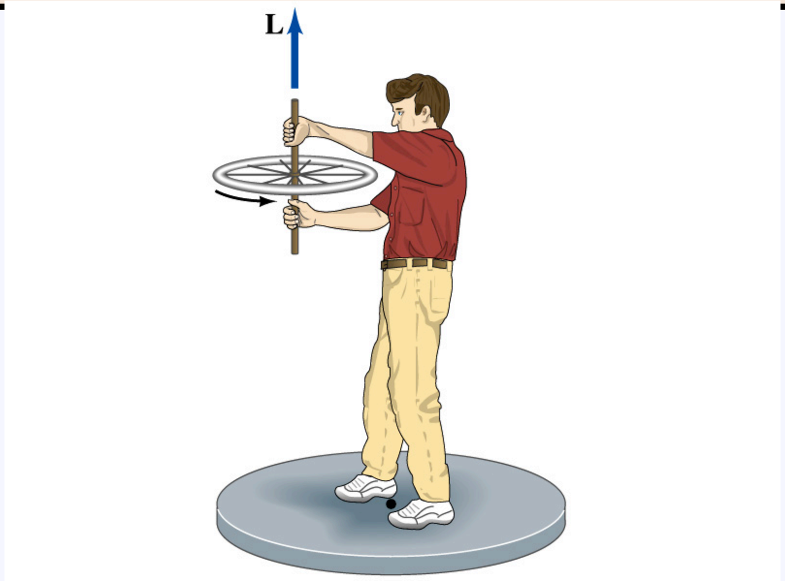
Question 8.12 Spinning Bicycle Wheel

You are holding a spinning bicycle wheel while standing on a stationary turntable. If you suddenly flip the wheel over so that it is spinning in the opposite direction, the turntable will:

A) remain stationary

B) spin in the same direction as the wheel (before flipping)

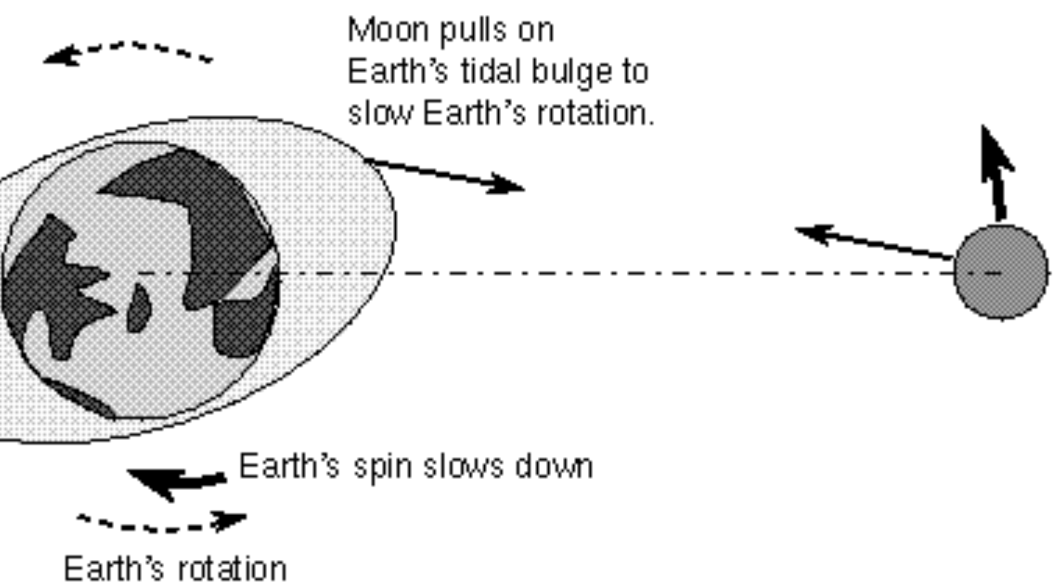
C) spin in the same direction as wheel (after flipped)



Clicker question #114

- The tidal interaction between the earth & the moon is decreasing Earth's angular velocity, making days slightly longer (21st century will be 25 s longer than 20th).

  What happens to moon's **angular momentum** as a result?



A Magnitude increases

B Magnitude decreases

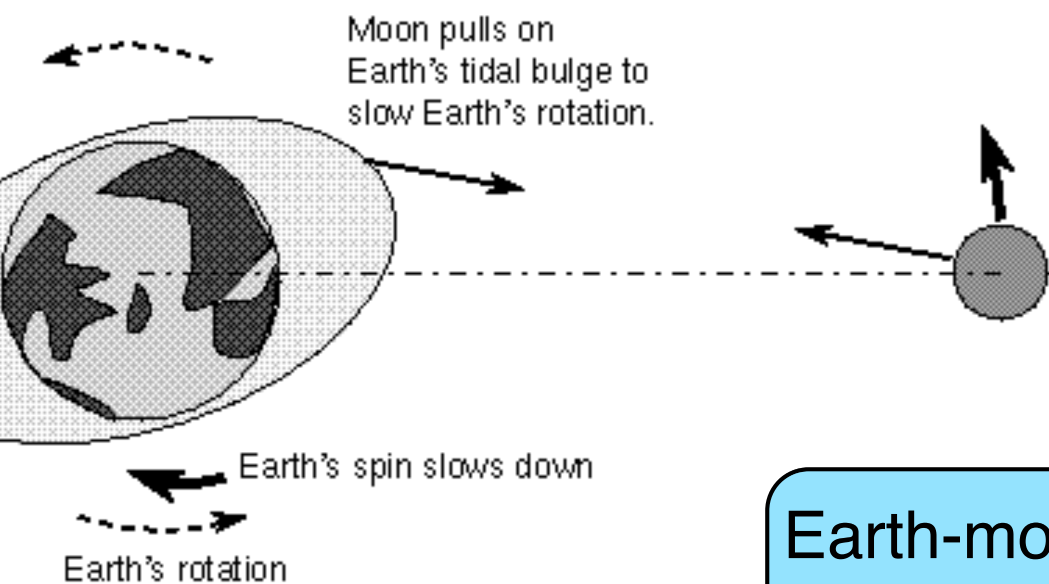
C Stays constant

D Changes direction

Clicker question #114

- The tidal interaction between the earth & the moon is decreasing Earth's angular velocity, making days slightly longer (21st century 25 s longer than 20th).

  What happens to moon's **angular momentum** as a result?



A Magnitude increases

B Magnitude decreases

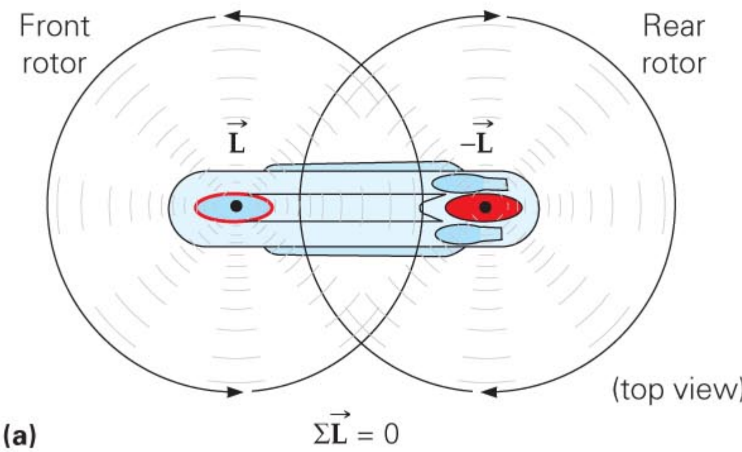
C Stays constant

D Changes direction

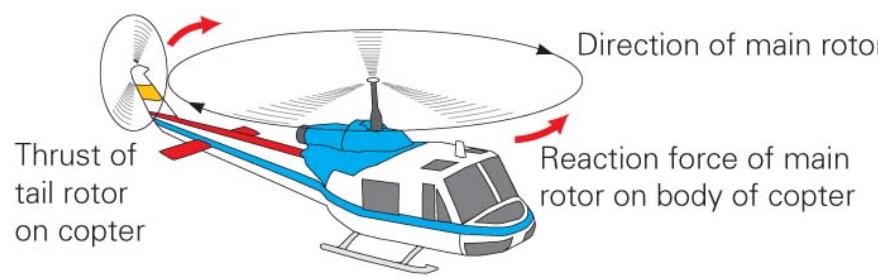
Earth-moon distance (≈400000 km) increases 4 cm/year

Angular momentum: helicopters

- Helicopter
  http://www.youtube.com/watch?v=nWk0jUfGrto

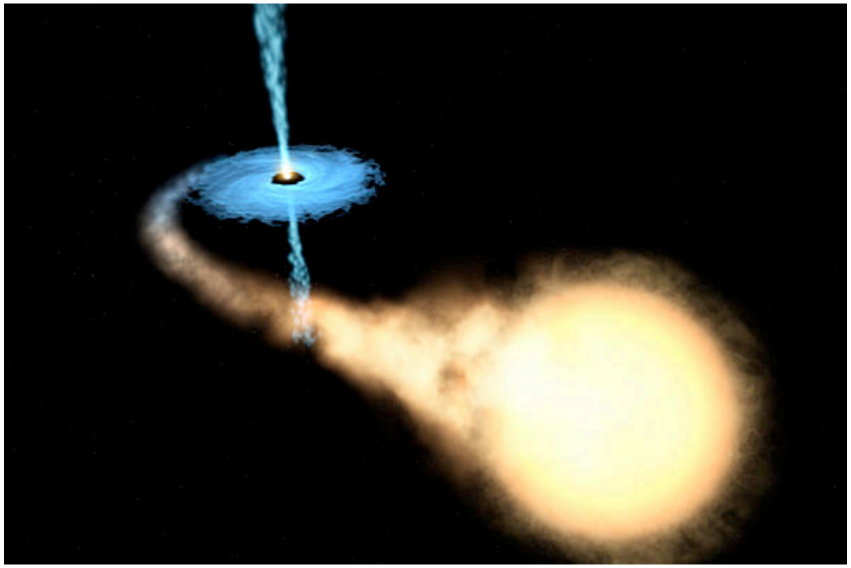


(a)



(b)

Angular momentum examples



- Spinning black holes
  - E.g. Cygnus X-1

$$L_{\mathrm{Cygnus X-1}} \approx 10^{44} \mathrm{kg} \cdot \mathrm{m}^2/\mathrm{s}$$

X-rays from matter falling into Cygnus X-1

- Electrons: intrinsic angular momentum ("spin") (chemistry) $L_{\mathrm{ElectronSpin}} \approx 10^{-34} \mathrm{kg} \cdot \mathrm{m}^2/\mathrm{s}$

Lecture 24 outline

- Announcements
- Angular momentum
  - Definition
  - Conservation
  - Examples & demos
- Bonus examples
  - Rotational energy / power / work
  - Torque
  - Rotational dynamics

Example: king of torque

- The Chevy “King of Torque” Silverado outputs 650 ft•lb = 880 m•N of torque at 1600 rpm.
  a) What is the power output of the “360-horsepower” engine under these conditions?

  b) A torque of 880 m•N is applied to a wheel (moment of inertia 0.5 kg•m^2). What is its angular acceleration?

Example: king of torque

- The Chevy "King of Torque" Silverado outputs 650 ft•lb = 880 m•N of torque at 1600 rpm.
  a) What is the power output of the "360-horsepower" engine under these conditions?

Given: $\tau = 880 \text{ m} \cdot \text{N}$
$\omega = 1600 \text{ rpm} = 170 \text{ rad/s}$

Goal: $P = ?$

Principle & eqns.

$$P = \tau\omega$$

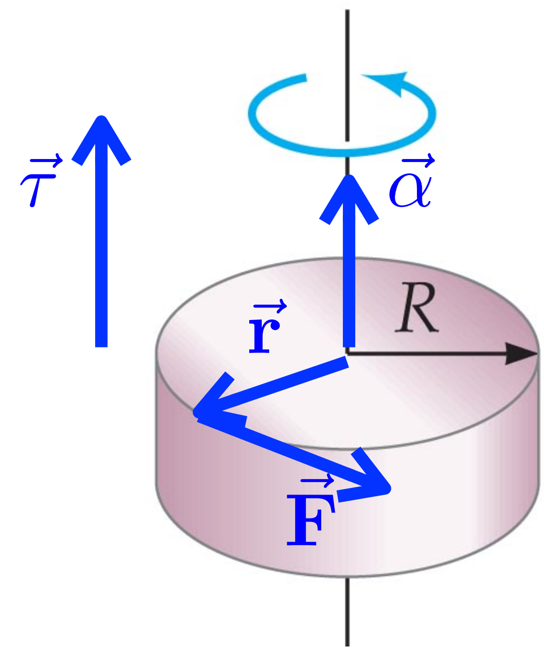
$$P = (880 \text{ m} \cdot \text{N})\,(170 \text{ rad/s}) = 150 \text{ kW} = 200 \text{ hp}$$

Example: king of torque

- b) A torque of 880 m•N is applied to a wheel (moment of inertia 0.5 kg•m^2). What is its angular acceleration?

Given: $\tau = 880 \text{ m} \cdot \text{N}$
$I = 0.5 \text{ kg} \cdot \text{m}^2$

Goal: $\alpha = ?$



Principle & eqns.

Newton's second law (rotational)

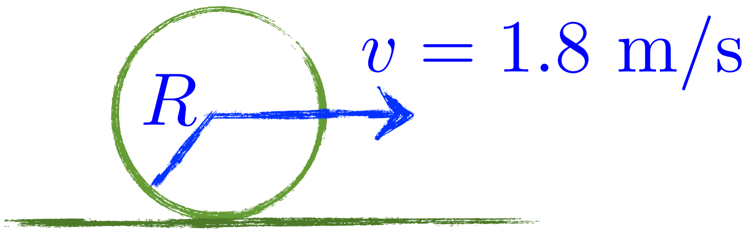
$\vec{\tau} = I\vec{\alpha}$

$$\alpha = \frac{\tau}{I} = \frac{880 \text{ m} \cdot \text{N}}{0.5 \text{ kg} \cdot \text{m}^2} = 1800 \text{ rad/s}^2$$

Ex. 8.15

- A uniform, solid cylinder (mass = 1.0 kg) rolls without slipping (speed = 1.8 m/s) on a level surface. What fraction of the total kinetic energy is rotational?

$M = 1.0\ \mathrm{kg}$



$I = \frac{1}{2}mR^2$

Given: $M = 1.0\ \mathrm{kg}$ $\quad v = 1.8\ \mathrm{m/s}$

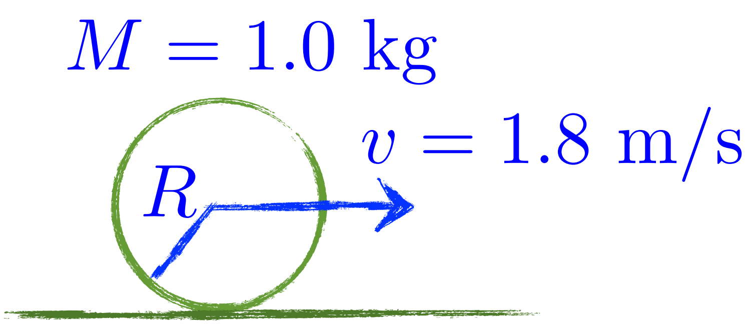
Goal: $K_{\mathrm{rot}}/(K_{\mathrm{rot}} + K_{\mathrm{trans}})$

Principles & equations:

$$K_{\mathrm{rot}} = \frac{1}{2}I\omega^2 \quad K_{\mathrm{trans}} = \frac{1}{2}mv^2$$

$$v = R\omega \text{ (roll without slipping)}$$

Ex. 8.15



$$I = \frac{1}{2}mR^2$$

Given: $M = 1.0\ \mathrm{kg} \quad v = 1.8\ \mathrm{m/s}$

Goal: $K_{\mathrm{rot}}/(K_{\mathrm{rot}} + K_{\mathrm{trans}})$

Principles & equations:

$$K_{\mathrm{rot}} = \frac{1}{2}I\omega^2 \quad K_{\mathrm{trans}} = \frac{1}{2}mv^2$$

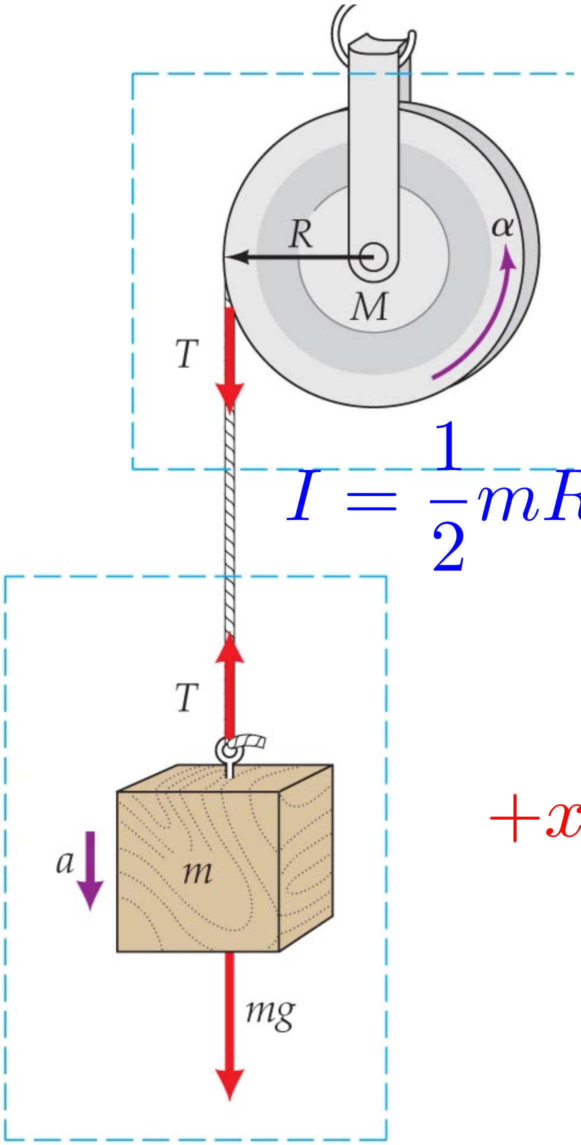
$$v = R\omega \text{ (roll without slipping)}$$

$$\frac{K_{\mathrm{rot}}}{K_{\mathrm{rot}} + K_{\mathrm{trans}}} =$$

$$\frac{\frac{1}{2}I\omega^2}{\frac{1}{2}I\omega^2 + \frac{1}{2}mv^2} = \frac{I\omega^2}{I\omega^2 + mR^2\omega^2} = \frac{I\omega^2}{I\omega^2 + 2I\omega^2} = \frac{1}{1+2} = \frac{1}{3}$$

Ex. 8.13

- A mass *m* hangs from a pulley of mass *M* and radius *R*. What is the linear acceleration of the mass?

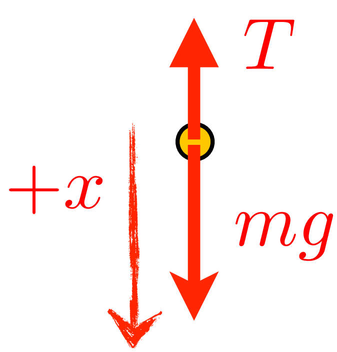


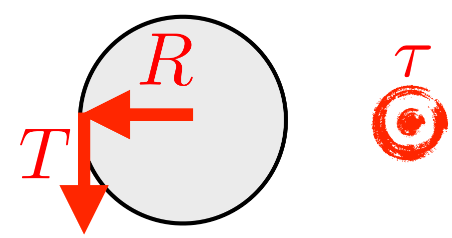
$$I = \frac{1}{2} m R^2$$

Given: mass *m*, pulley mass *M*, radius *R*

Goal: block acceleration *a*

Principle & eqns.: Newton's second law, free body diagrams, rolling without slipping



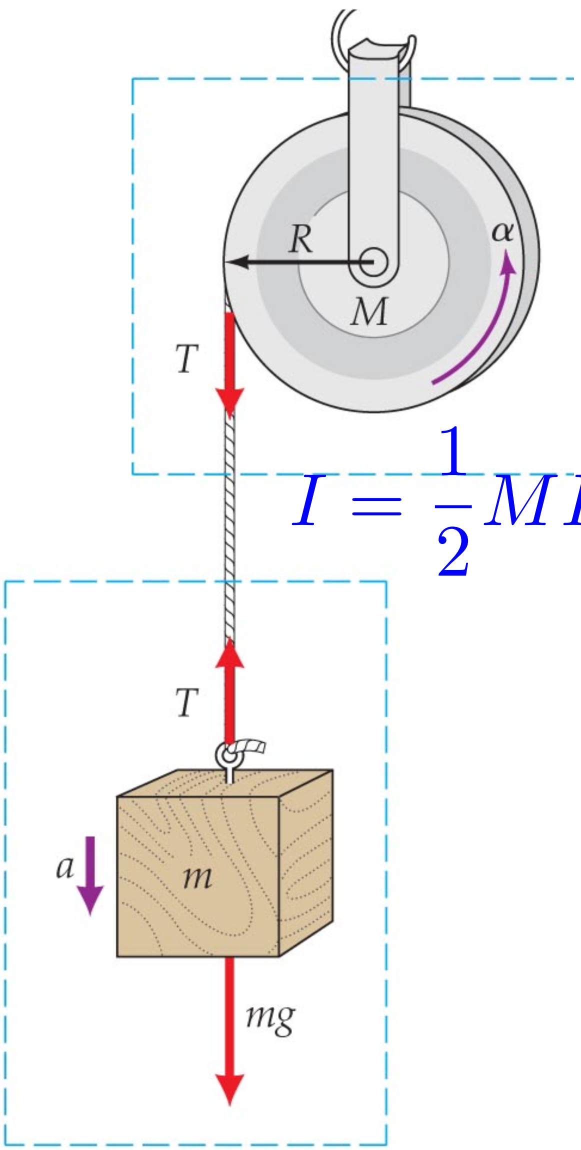


Ex. 8.13

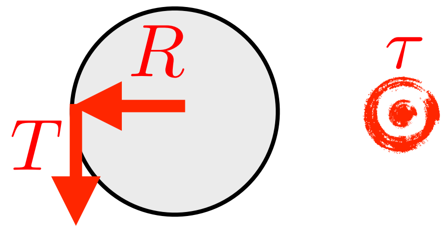
Given: mass *m*, pulley mass *M*, radius *R*

Goal: block acceleration *a*

Principle & eqns.: Newton's second law, free body diagrams, tangential accel.



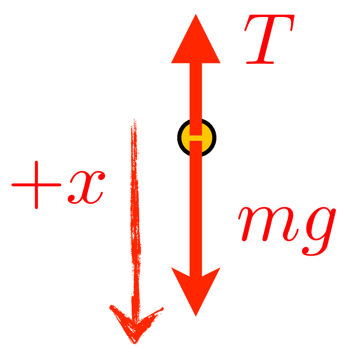
$$I = \frac{1}{2}MR^2$$



$$a = R\alpha$$

$$RT = I\alpha = \frac{1}{2}MR^2\frac{a}{R}$$

$$T = \frac{1}{2}Ma$$



$$mg - T = ma$$

$$mg - \frac{1}{2}Ma = ma$$

$$a\left(m + \frac{1}{2}M\right) = mg$$

$$a = \frac{m}{m + (1/2)M}g$$

Class participation #23

- 0. Full name
- 1. Like best about the class?
- 2. Like least about the class?