Physics 2213a, Modern Physics **Professor Rick Trebino**

Problem Set #3 **Special Relativity**

Note that there are answers to many Thornton and Rex problems in the back of the book, so, if you like, you can do lots of practice problems whose answers you can look up.

1. Determine the ratio $\beta = v/c$ for the following:
   (a) A race car traveling 200 km/h.
   (b) A commercial jet airliner traveling 300 m/s.
   (c) A supersonic airplane traveling at Mach 2.3
   (the Mach number = u/v_{sound}).
   (d) The space shuttle, traveling 27,000 km/h.
   (e) An electron in a beam of electrons traveling 25 cm in 2 ns.
   (f) A proton traveling across a nucleus (10^{-14} m) in 0.35×10^{-22} s.

   Find the relativistic factor γ for each of the above objects. If γ is too close to 1 for your calculator, just provide $\gamma - 1$.

2. Two events occur in an inertial system K as follows:
   Event 1: $x_1 = a_1$ $t_1 = 2a/c$ $y_1 = 0$ $z_1 = 0$
   Event 2: $x_2 = 2a_1$ $t_2 = 3a/2c$ $y_2 = 0$ $z_2 = 0$
   In what frame K′ will these events appear to be simultaneous? Is there a frame K′ in which the two events described above occur at the same place? Explain.

3. Astronomers discover a planet orbiting around a star similar to our sun 20 light-years away. How fast must a rocket ship go if the round trip is to take 40 years in time for the astronauts abroad? How long will the trip take as measured on Earth? (Assume a constant velocity, and don't worry about accelerating or decelerating.)

4. A proton and an antiproton are moving toward each other in a head-on collision. If each has a speed of $0.8c$ with respect to the collision point, how fast are they moving with respect to each other?

5. An astronaut tries to talk his way out of a traffic violation for running a red light (λ = 670 nm) by telling the judge that the light appeared green (λ = 540 nm) to him as he passed by in his expensive sports car. If this is true, how fast was the astronaut going? Will this excuse get him out of trouble?

6. The Lockheed SR-71 Blackbird may be the fastest non-research airplane ever built; it traveled at 2200 miles/hour (983 m/s) and was in operation from 1966 to 1990. Its length is 32.74 m.
(a) By what percentage would it appear to be length contracted while in flight?
(b) How much younger would a pilot who took 1000 flights in the plane be compared to his twin on Earth if each of his flights on the Blackbird covered 3200km?

7. Quasars are among the most distant objects in the universe and are receding from us at the highest known speeds. Astrophysicists use the redshift parameter z to determine the speed of such rapidly moving objects. The parameter z is determined by observing a wavelength λ' of a known spectral line of wavelength λ_{source} on Earth:

$$z = \Delta\lambda / \lambda_{source} = (\lambda' - \lambda_{source}) / \lambda_{source}$$

Find the recession velocities of two quasars having z values of 1.9 and 4.9.

8. Suppose that a gamma-ray burster (traveling at speed v) shines a beam of light at an angle θ' to its direction of travel in its reference frame. Derive an expression for the angle, θ, of this beam of light in a non-moving reference frame. Hint: it's best to compute the tangent of θ. Also the x' and y' components of the beam's velocity relative to the burster in its frame are: $u_x' = c \cos\theta'$ and $u_y' = c \sin\theta'$.

Evaluate your result for the case that v ≈ c and θ' = 90°, and show that, when v approaches c, most of the light emitted by any object is "beamed" into the forward direction—even if the light is emitted in all directions in its frame!

9. In hydrogen-bomb (nuclear fusion) explosions, blasts of as much as 100 Megatons (of TNT) have been recorded. For your information, a Megaton is the energy that would be released if 1,000,000 tons of TNT had exploded, and 1g of TNT releases 1000 calories (or 4180J) of energy. How much mass (of hydrogen) is converted to energy to yield a 100 Megaton blast? Because hydrogen is the most plentiful atom in the universe and on our planet, you can see why humankind is putting so much effort into trying to make nuclear fusion into a peaceful energy source.

10. If you're into mega-disasters (and who isn't?), consider this one. The universe most likely began with equal amounts of matter and anti-matter, which annihilate in a spectacular explosion to produce pure energy when they collide with each other. Fortunately, for reasons no one understands yet, most anti-matter has disappeared from at least our neighborhood of the universe. But suppose that an anti-matter asteroid crashed into the earth. If the asteroid were spherical and only 100m across, with a density of $2 g/cm^3$, to how many 100 Megaton nuclear blasts would its collision with earth be equivalent? Does it matter how fast it's traveling relative to earth?