Enderton 1.1, Language of Sentential Logic

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1 At the Zoo

When you visit a zoo, there are questions that always get asked. Many of these apply also to mathematical objects.
Zoo questions of today's objects:

- Where does it live? How does it function in its environment?
- What body parts does it have?
- What symmetries does it have?
- How fast does it run?
- How does it reproduce/grow? How fast?
- How are they usefully analyzed?
- What does it eat?
- What are its relatives?
- What powers and weaknesses does it have, and how do they trade off?
- What are non-examples?

2 Example

Example:

$$((A \wedge B) \to (\neg C)).$$

What are syntactical features?
What are semantic features (for next time)?

3 Main ideas

- Symbols of sentential logic are (p.14) $(,), \neg, \wedge, \vee, \to, \leftrightarrow, \mathbf{A}_1, \mathbf{A}_2, \ldots$
- The well-formed formulae, wffs, are given by a grammar (formula-building rules):
$$S : (\neg S)|(S \wedge S)|(S \vee S)|(S \to S)|(S \leftrightarrow S)|\mathbf{A}_1| \ldots.$$
- How to work with wffs?
  1. Parse ("Ancestral") trees
  2. Construction sequences
  3. Use induction.
     Induction principle (structural induction), proved by induction on length of construction sequence, height of the parse tree, or otherwise. We can read "If a set S of wffs..." as "If a property P of wffs

4 Zoo Answers

Some zoo answers follow. "It" is a well-formed formula of sentential logic.

- Where does it live? How does it function in its environment? A wff is a formal mathematical counterpart of an English sentence (select sentences only). It is found where we want a formal declarative sentence.
- What body parts does it have? A wff is formed from $(,), \neg, \wedge, \vee, \to, \leftrightarrow, \mathbf{A}_1, \mathbf{A}_2, \ldots$
- What symmetries does it have? Balanced parentheses, self-similarity of construction
- How fast does it run? (We'll discuss algorithms shortly.)
- How does it repoduce/grow? How fast? A wff is constructed from other wffs by the formula-building operations. In n steps, a wff grows to size about n and height log(n) to n.
- How are they usefully analyzed? Induction (many forms), on the construction sequence and ancestral tree.
- What does it eat? (Best answered semantically, shortly.)
- What are its relatives? What are non-examples? We will consider first-order logic and (briefly) higher-order and modal logics later. These (and not sentential wffs) have quantifiers, functions, relations, and more.
- What powers and weaknesses does it have, and how do they trade off? These wffs are relatively weak in expressive power, but relatively straightforward to analyze. Details over the next few weeks.