Math 481 Homework 1

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Due September 21, 2011, at start of class

Sample symbols: $\alpha \subsetneq \beta$.

1. (5 pts.; Baseed on Enderton 1.1.3.) Let α be a wff; let c be the number of places at which the four *binary* connective symbols $(\wedge, \vee, \rightarrow, \leftrightarrow)$ occur in α; let s be the number of places at which sentence symbols occur in α. (For example, if α is $(\mathbf{A} \rightarrow (\neg\mathbf{A}))$ then $c = 1$ and $s = 2$.) Show by using the induction principle that $s = c + 1$.

2. (5 pts.; Based on Enderton 1.2.1.) Show that neigher of the following two formulas tautologically implies the other:

$$(\mathbf{A} \leftrightarrow (\mathbf{B} \leftrightarrow \mathbf{C}))$$
$$((\mathbf{A} \wedge (\mathbf{B} \wedge \mathbf{C})) \vee ((\neg\mathbf{A}) \wedge ((\neg\mathbf{B}) \wedge (\neg\mathbf{C}))))$$

   Suggestion: Only two truth assignments are needed, not eight.

3. (5 pts.; Based on Enderton 1.2.4a.) Show that $\Sigma; \alpha \models \beta$ iff $\Sigma \models (\alpha \rightarrow \beta)$. Show that, for *finite* Σ, $\Sigma \models \beta$ iff $\models ((\bigwedge \Sigma) \rightarrow \beta)$.

   Suggestion: Use induction on the size of Σ.

   Recall that $\Sigma; \alpha$ means $\Sigma \cup \{\alpha\}$ and $\bigwedge \Sigma$ means $(\sigma_1 \wedge (\sigma_2 \wedge (\cdots \wedge \sigma_n)) \cdots)$, where $\Sigma = \{\sigma_1, \sigma_2, \ldots, \sigma_n\}$.

4. (5 pts.; Based on Enderton 1.3.4.) Suppose that we modify our definition of wff by omitting all *right* parentheses. Thus instead of

$$((\mathbf{A} \wedge (\neg\mathbf{B})) \rightarrow (\mathbf{C} \vee \mathbf{D}))$$

   we use

$$((\mathbf{A} \wedge (\neg\mathbf{B} \rightarrow (\mathbf{C} \vee \mathbf{D}.$$

   Show that we still have unique readability (i.e., each wff still has only one possible derivation over the grammar gotten by removing right parentheses from our original grammar.)