Math 481 Homework 4

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Due Friday, October 12, 2012, at start of class

1. In this problem, we will reduce the solubility of a sudoku to a bike tangle.

   (a) Fix a row r and cells c_1, c_2, c_3 in r. Among other things, we may want to express that c_1 is assigned a 1 or a 2 but not both and, if cell c_2 is assigned a 1 or cell c_3 is assigned a 1, then cell c_1 is not assigned a 1. Construct a ϕ that is satisfiable iff the constraints can be satisfied simultaneously. More generally, Describe a way, given an $n \times n$ sudoku S, to produce a cnf φ, such that φ is satisfiable iff S has a solution.

   (b) Describe a way, given a CNF φ, to construct a tangle t of bike locks and a number k, such that opening k locks frees the bike in t iff φ is satisfiable. *Hint: for each sentence symbol in φ, say, **A**, put two locks into t, one marked **A** and one marked $(\neg \mathbf{A})$. Think of φ as monotonic over literals.*

   Further instructions: The tangle and formula you construct should demonstrate the general simluation procedure; components of tangles or formulas should correspond to constraints and it should be obvious how to build graphs quickly from the set of constraints. There is no need to justify correctness of ϕ or t, but correctness should be fairly obvious.

   To ease grading, please use the following semantics for sentence symbols:

Sentence Symbol	Semantics
A	c_1 gets 1
B	c_2 gets 1
C	c_3 gets 1
D	c_1 gets 2

2. (5 pts.; Based on Enderton 1.3.4.) Suppose that we modify our definition of wff by omitting all *right* parentheses. Thus instead of

$$((\mathbf{A} \wedge (\neg \mathbf{B})) \rightarrow (\mathbf{C} \vee \mathbf{D}))$$

   we use

$$((\mathbf{A} \wedge (\neg \mathbf{B} \rightarrow (\mathbf{C} \vee \mathbf{D}.$$

Show that we still have unique readability (i.e., each wff still has only one possible derivation over the grammar gotten by removing right parentheses from our original grammar.)