

Math 53: Lec 002 Sec 213

Triple Integral

18 October 2012

Learning Objectives: 1) Different kinds of triple integral; 2) The relationship between triple and double integral; 3) Triple integral as volume; 4) Center of mass of a solid.

I. Reading Quiz

1. What are the three different types of triple integral over a general bounded region?

2. How would you conceptualize a triple integral? Hint: what if the function is 1?

3. What is the formula for the coordinates $(\bar{x}, \bar{y}, \bar{z})$ of the center of mass of a solid occupying the region E and having density function $\rho(x, y, z)$? What is the mass m given by?

4. Evaluate $\iiint_E 8xyz \, dV$, where $E = [2, 3] \times [1, 2] \times [0, 1]$.

II. Problems

1. Evaluate $\iiint_E 2xe^y \sin z \, dV$, where E is the rectangle defined by

$$Q = \{(x, y, z) \mid 1 \leq x \leq 2, 0 \leq y \leq 1 \text{ and } 0 \leq z \leq \pi\}$$

Each person should pick a different order of integration and show that you get the same result.

2. Set up and evaluate a triple integral to find the volume of a unit sphere. You should be able to check your answer.

3. Write five other iterated integrals that are equal to $\int_0^1 \int_y^1 \int_0^y f(x, y, z) \, dx \, dy \, dz$.

4. Find the center of mass of the solid of a constant mass density ρ bounded by $z = \sqrt{x^2 + y^2}$ and the plane $z = 4$.