# Week 6: §6.2–§6.4

## Binomial Distributions

Suppose we have a binomial experiment with $n$ trials. We can define a random variable $X$ to be the number of successes, so $X$ ranges from 0 to $n$.

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**Definition:** For a given $n$ and $p$, let $b(k) = C(n,k)p^k q^{n-k}$, the probability of getting exactly $k$ successes. We sometimes write $b(k;n,p)$ when we want to emphasize or clarify the values of $n$ and $p$. The probability distribution of the associated random variable $X$ is called the ***binomial distribution***.

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Recall the coffee problem from the previous activity. There, you were asked to determine the probability that at most three students out of 20 prefer coffee. Getting a numerical answer for such a question is tedious, so we have binomial distribution tables that allow us to look up the numerical answers for many such problems.

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**Definition:** For a given $n$ and $p$, let $B(x) = b(0)+b(1)+\cdots+b(x)$, the probability of getting at most $x$ successes. We sometimes write $B(x;n,p)$ when we want to emphasize or clarify the values of $n$ and $p$. This is called the ***cumulative binomial distribution function***. The table in Appendix A gives values of $B$ for various values of $x$, $n$, and $p$.

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**1.** Use the table in Appendix A to find the probability of getting at most five successes in a binomial experiment with eleven trials and a probability of success $p = 0.25$.

$$B(5; 11, 0.25) = 0.9657$$

**2.** Use the table to find the probability of getting exactly five successes in a binomial experiment with eleven trials and a probability of success $p = 0.25$. (Hint: think of how $B$ is defined in terms of $b$.)

$$B(5; 11, 0.25) - B(4; 11, 0.25) = 0.9657 - 0.8854$$

**3.** Write $b(x)$ in terms of the function $B$.

$$b(x) = B(x) - B(x-1)$$

**4.** The table gives us probabilities of getting *at most* a certain number of successes. How can we use it to find the probability of getting at *least* a specified number of successes, for example at least three successes in seven trials with $p = 0.35$?

$$b(3) + \dots + b(7) = 1 - (b(0) + b(1) + b(2))$$
$$= 1 - B(2; 7, 0.35)$$
$$= 1 - 0.5323$$

**5.** The values of $p$ in the table are no greater than 0.5. How can we use the table to find, for example, $B(3; 9, 0.6)$?

Switch the roles of success and failure, so $B(3; 9, 0.6)$ is the same as the probability of getting at least 6 failures ($q = 0.4$), which as in #4 is $1 - B(5; 9, 0.4)$, or $1 - 0.9006$.

**6.** Find the probability of getting at least five successes in a binomial experiment with eleven trials and a probability of success $p = 0.25$.

$$1 - B(4; 11, 0.25)$$
$$= 1 - 0.8854$$

**7.** Find the probability of getting at least five successes in a binomial experiment with eleven trials and a probability of success $p = 0.7$.

This is the probability of getting at most six failures ($q = 0.3$), which is $B(6; 11, 0.3) = 0.9784$.

## Normal Distributions

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**Definition:** A ***continuous random variable*** is one that can take on any value on some interval of real numbers.

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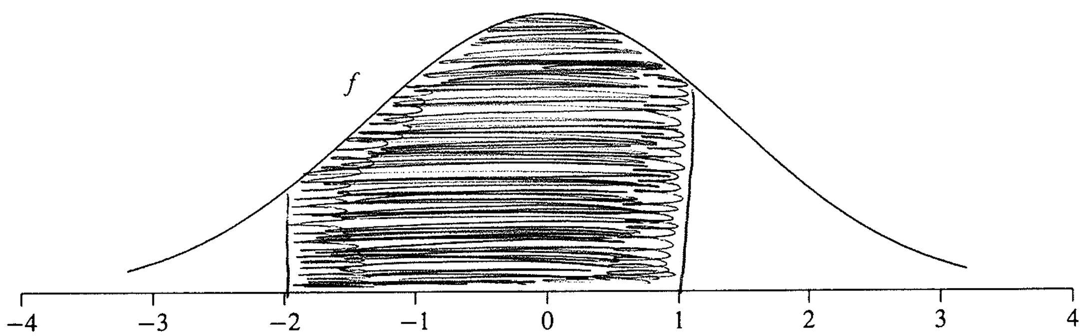
Recall that a histogram is a visual representation of a random variable, and that the total area under all of the bars is 1. With a continuous random variable, instead of bars we use a ***probability density function***, the area under whose graph is 1.

**1.** In the following graph of a probability density function $f$ of a random variable $X$, shade the area corresponding to $X$ being between $-2$ and 1.

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**Definition:** The ***standard normal distribution***, whose associated random variable we denote by $Z$, is given by a curve like that above such that the mean $\mu$ is 0 and the standard deviation $\sigma$ is 1.

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**2.** Use the table in Appendix B to find the following probabilities.

**a)** $\Pr(Z \leq 2.1) = 0.9821$

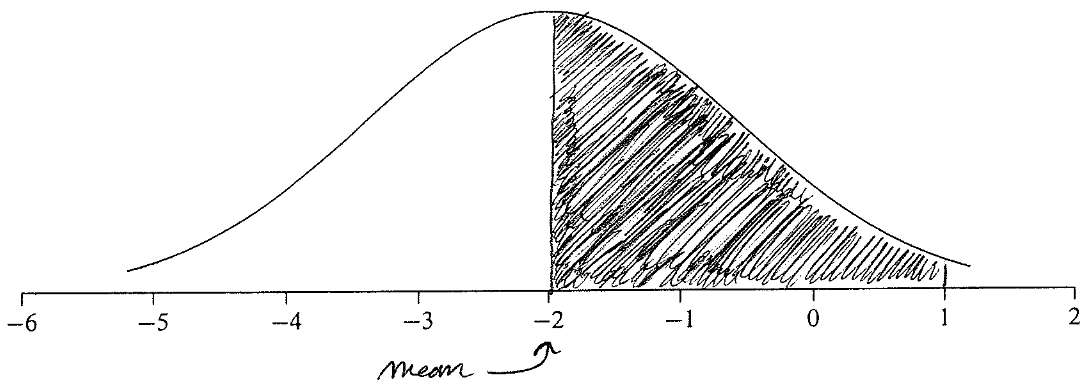
**b)** $\Pr(Z \leq 2.14) = 0.9838$

**c)** $\Pr(-2 \leq Z \leq 1) = \Pr(Z \leq 1) - \Pr(Z \leq -2) = 0.8413 - 0.0228$

**d)** $\Pr(Z \geq 0.5)$ $1 - \Pr(Z \leq 0.5) = 1 - 0.6915$

**Definition:** A ***normal distribution*** is determined by a curve like the one that determines the standard normal distribution, but we are now free to change the mean $\mu$ and standard deviation $\sigma$.

**3.** Suppose that the following graph is a normal distribution determined by a random variable $X$. Label the mean, and shade the area corresponding to $X$ being between $-2$ and 1.




**Remark:** We can use the same table in Appendix B to find values like $\Pr(X \leq 0)$ by using the following facts. If $X$ has mean $\mu$ and standard variation $\sigma$, then

$$Z = \frac{X-\mu}{\sigma},$$

which means that

$$\Pr(a \leq X \leq b) = \Pr\left(\frac{a-\mu}{\sigma} \leq Z \leq \frac{b-\mu}{\sigma}\right)$$

**4.** Suppose that $X$ is a normally distributed random variable with $\mu = 2$ and $\sigma = 0.5$. Use the table in Appendix B to find the following probabilities.

a) $\Pr(X \leq 2.1)$ $\Pr\left(Z \leq \frac{2.1-2}{0.5}\right) = \Pr(Z \leq 0.2) = 0.5793$

b) $\Pr(X \leq 2.14)$ $\Pr\left(Z \leq \frac{2.14-2}{0.5}\right) = \Pr(Z \leq 0.28) = 0.6103$

c) $\Pr(1 \leq X \leq 2.1)$ $\Pr(Z \leq 0.2) - \Pr\left(Z \leq \frac{1-2}{0.5}\right) = 0.5793 - \Pr(Z \leq -2)$

$= 0.5793 - 0.228$

d) $\Pr(X \geq 0.5)$ $1 - \Pr(X \leq 0.5)$

$= 1 - \Pr\left(Z \leq \frac{0.5-2}{0.5}\right)$

$= 1 - \Pr(Z \leq -3) = 1 - 0.0013$