Lecture 12: Stress, Strain, and Deflection II

BAEN 375

Design Fundamentals of Agricultural Machines and Structures

Strain

- Strain is related to stress by Hooke's law in the elastic region.
- Strain is also a tensor and can be represented by the following matrices for 3D and 2D cases, respectively:

$$\begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz} \end{bmatrix}$$

$$\begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} \\ \varepsilon_{yx} & \varepsilon_{yy} \end{bmatrix}$$

- ε can represent normal or shear strain. Subscripts determine.
- As with stress, we simplify ε_{xx} as ε_x, and ε_{xy} = ε_{yx}

Stress

- There will be planes on which shear stress components are zero
- These planes are called **principal planes**
- The normal stresses acting on the principal planes are called **principal normal stresses**
- The surface normals act in the directions of the **principal axes**
- The **principal shear stresses** act on planes at 45° to the principal planes

Stress

- We are concerned about failure of machine parts
- So we must find the largest **normal and shear** stresses occurring in a part
- If the material can be considered isotropic, we need not worry about the direction of these principal stresses

Stress

- Applied stresses are related to principal stresses by the following equation:

$$\begin{bmatrix} \sigma_x - \sigma & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y - \sigma & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z - \sigma \end{bmatrix} \begin{bmatrix} n_x \\ n_y \\ n_z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

- If this equation is solvable, then the determinant of the coefficient matrix must be zero, so we have the cubic polynomial

$$\sigma^3 - C_2\sigma^2 - C_1\sigma - C_0 = 0$$

- Where

$$C_2 = \sigma_x + \sigma_y + \sigma_z$$

$$C_1 = {\tau_{xy}}^2 + {\tau_{yz}}^2 + {\tau_{zx}}^2 - \sigma_x\sigma_y - \sigma_y\sigma_z - \sigma_z\sigma_x$$

$$C_0 = \sigma_x\sigma_y\sigma_z + 2\tau_{xy}\tau_{yz}\tau_{zx} - \sigma_x{\tau_{yz}}^2 - \sigma_y{\tau_{zx}}^2 - \sigma_z{\tau_{xy}}^2$$

- And

  $\sigma_1, \sigma_2, \sigma_3$ are roots of the cubic polynomial above

Stress

- The principal normal stresses $\sigma_1, \sigma_2, \sigma_3$ are
  - Always real
  - Usually ordered such that $\sigma_1 > \sigma_2 > \sigma_3$
  - Orthogonal
- The principal shear stresses are found by

$$\tau_{13} = \frac{|\sigma_1 - \sigma_3|}{2}$$

$$\tau_{21} = \frac{|\sigma_2 - \sigma_1|}{2}$$

$$\tau_{32} = \frac{|\sigma_3 - \sigma_2|}{2}$$

Stress

- In two dimensions, the principal normal stresses $\sigma_a, \sigma_b, \sigma_c$ are found by

$$\sigma_a, \sigma_b = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

$$\sigma_c = 0 = \sigma_2$$

- And

$$\tau_{max} = \tau_{13} = \frac{|\sigma_1 - \sigma_3|}{2}$$

Mohr's Circles

Allow graphical solution for principal stresses (mainly for two dimensions).

Procedure

(based on Ex. 4-1 in Norton)

1. Construct σ axis (x)
2. Construct τ axis (y)
3. Draw applied stress σ_x (positive = tensile in Ex. 4-1)
4. Draw applied stress σ_y (negative = compressive in Ex. 4-1)
5. Draw shear stress τ_{xy}
6. Draw shear stress τ_{yx}
7. Draw line (CD) connecting tips of τ_{xy} and τ_{xy}
8. Draw a Mohr's circle using intersection of lines CD and AB as center of circle
9. Two of three principal stresses are found where this circle intersects σ axis (σ_1 and σ_3). Since we are in two dimensions, $\sigma_2 = 0$.
10. Draw two more Mohr's circles.
11. Draw tangents from the circles to the τ axis. These tangents give the principal shear stresses. Typically we only care about which one of these when designing?