Lecture 22: Static Failure Theories IV

BAEN 375

Design Fundamentals of Agricultural Machines and Structures

Failure in Brittle Materials Under Static Load

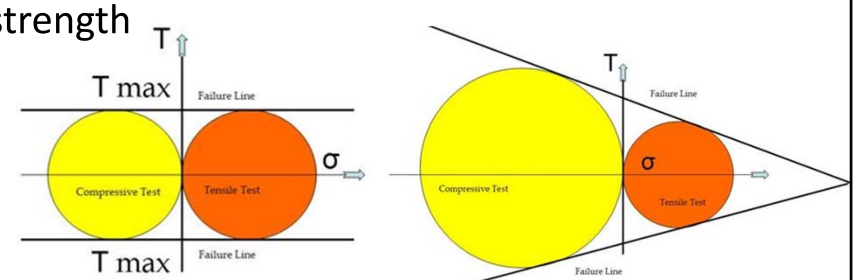
- Brittle Materials
  - Fracture instead of yield
  - Fail in tension (generally)
  - Fail due to normal tensile stress
  - Maximum Normal Stress Theory is applicable (modified version)

Failure in Brittle Materials Under Static Load

- Failure of brittle materials in compression is due to combined normal compressive stress and shear
- Some wrought metals can be brittle but even, but most brittle materials have compressive strengths much greater than their tensile strength
- This is due to the presence of microscopic flaws that serve as nuclei for crack formation under tensile loads

Failure in Brittle Materials Under Static Load

- Some cast, brittle materials have shear strength greater than tensile strength but less than compressive strength
- This is different from ductile materials, for which shear strength is about ½ of tensile strength



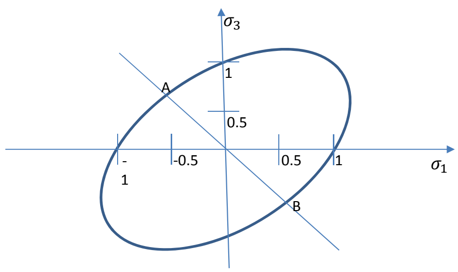
For a even Material : Ultimate compressive strength = Ultimate tensile strength

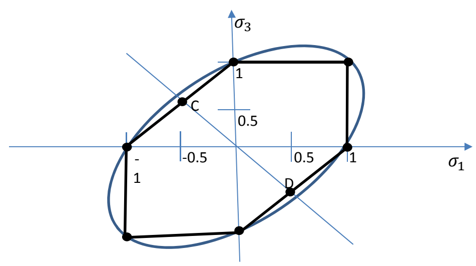
For an uneven Material : Ultimate compressive strength >> Ultimate tensile strength

Failure in Brittle Materials Under Static Load

- Remember that we had a failure ellipse determined from **Distortion-Energy Theory.**

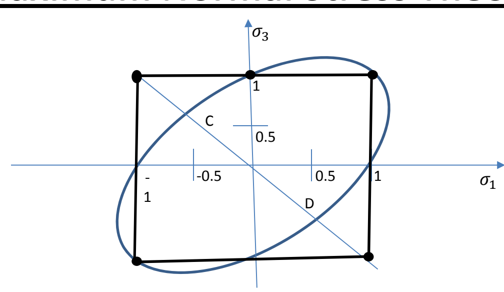
That theory was modified by the **Maximum Shear Theory**.





Failure in Brittle Materials Under Static Load

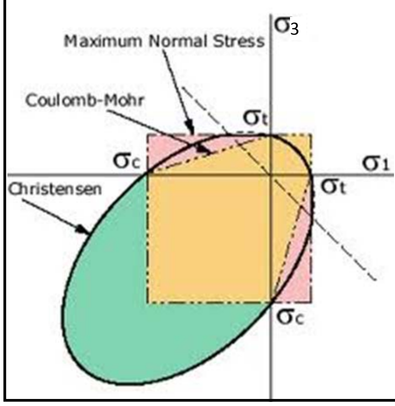
- With **Maximum Normal Stress Theory**, we have

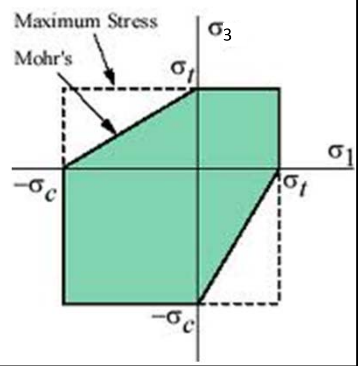


which we know doesn't work for ductile materials.

Failure in Brittle Materials Under Static Load

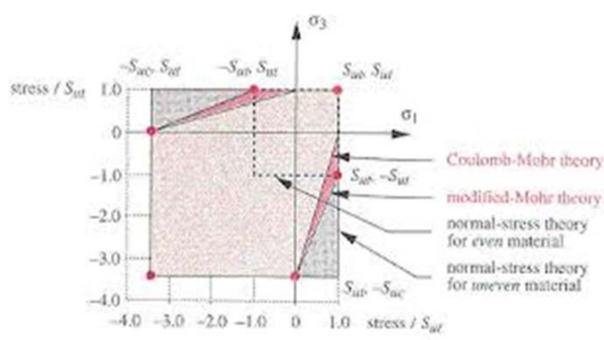
- But for brittle (uneven) materials, we have Coulomb-Mohr Theory





Failure in Brittle Materials Under Static Load

- Coulomb-Mohr Theory is modified to fit the actual data.

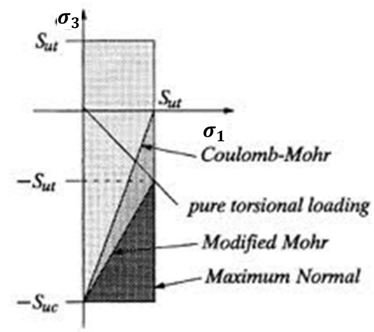


Failure in Brittle Materials Under Static Load

- If $\sigma_1 > \sigma_3$ and $\sigma_2 = 0$, we only need be concerned with quads 1 and 4.
- If σ_1 and σ_3 are positive,

$$N = \frac{S_{ut}}{\sigma_1}$$

- If σ_1 and σ_3 are opposite sign, then either $N = \frac{S_{ut}}{\sigma_1}$ or $N = \frac{S_{ut}|S_{uc}|}{|S_{uc}|\sigma_1 - S_{ut}(\sigma_1 + \sigma_3)}$
- Check both and use the smaller.



Failure in Brittle Materials Under Static Load

- Remember that we had Von Mises Effective Stress (σ') that accounted for overall stresses in ductile materials.
- We can have the same for brittle, $\tilde{\sigma}$, where:

$$C_1 = \frac{1}{2}\left[|\sigma_1 - \sigma_2| + \frac{2S_{ut} - |S_{uc}|}{-|S_{uc}|}(\sigma_1 + \sigma_2)\right]$$

$$C_2 = \frac{1}{2}\left[|\sigma_2 - \sigma_3| + \frac{2S_{ut} - |S_{uc}|}{-|S_{uc}|}(\sigma_2 + \sigma_3)\right]$$

$$C_3 = \frac{1}{2}\left[|\sigma_3 - \sigma_1| + \frac{2S_{ut} - |S_{uc}|}{-|S_{uc}|}(\sigma_3 + \sigma_1)\right]$$

Failure in Brittle Materials Under Static Load

- And

  $\tilde{\sigma}$ = effective stress for brittle (uneven) materials

  $\tilde{\sigma} = \max(C_1, C_2, C_3, \sigma_1, \sigma_2, \sigma_3)$

  $\tilde{\sigma} = 0$ if max < 0

  $N = \frac{S_{ut}}{\tilde{\sigma}}$