Handout 2

09.06.12

Topic: Force of Interest+Annuity+Perpetuities

Agenda:

1. Warm Up & Review with homework questions
2. Summary
3. Calculator requirement & Calculator methods review
4. Problems to try out

- Warm Up

  Recall: From Handout 1

  (3) AV of \$1,000 at the end of 6 years, at a nominal annual rate of discount of 5% compounded semiannually.

  Vote for hardness of HW…

  HW solution will be posted on Bspace. Save question for next class.

- Summary

  1. Force of interest

  a) Why do we need it?

  Effective rate cannot give us information about the fund at any moment of time.

  b) Definition

  The slope of the accumulation function divided by the amount in the fund at time t

  $\delta_t = \frac{1}{a(t)}\frac{d}{dt}a(t)$

  c) Related formula

Given force of interest	$a(t) = e^{\int_0^t \delta_r dr}$	$a(t) = e^{\delta t} = (1+i)^t$
Given interest rate (i)	$e^{\delta} = 1 + i$	$\delta = \ln(1+i)$

  d) Special interpretation

  Nominal rate of interest compounded continuously

$$\lim_{m\to\infty} i^{(m)} = \ln(1+i) = \delta$$

  e) Also useful to know

  Taylor expansion: $e^x = 1+x+x^2/2!+x^3/3!+\ldots \Rightarrow 1+x$

  $\ln(1+x) = x - x^2/2+x^3/3-\ldots \Rightarrow x$

  So approximately: $e^{\delta} = 1 + \delta$

$$\delta = i - \frac{i^2}{2} + \frac{i^3}{3} - \cdots = -\ln(1-d)$$

  2. Level Annuity

  a) Recall PV, FV Notice: $PV = FV * v^t$

  b) Geometric Sum = $(\text{first term})\frac{1-(ratio)^{no.of\ terms}}{1-ratio}$

c) (a) PV of an annuity-immediate:

$$a_{\overline{n}|} = v + v^2 + \cdots + v^n$$
$$= \frac{1 - v^n}{i}$$

(b) AV on the date of the last payment:

$$s_{\overline{n}|} = 1 + (1+i) + \cdots + (1+i)^{n-1}$$
$$= \frac{(1+i)^n - 1}{i}$$

d) PV/AV of annuity-due (change i to d) --- complete on your own

PV=

AV=

e)

Relationships between annuity-immediate and annuity-due:

$$\ddot{a}_{\overline{n}|} = (1+i)a_{\overline{n}|} = 1 + a_{\overline{n-1}|}$$
$$\ddot{s}_{\overline{n}|} = (1+i)s_{\overline{n}|} = s_{\overline{n+1}|} - 1$$

d) Trick to remember: use "i" for "immediate", "d" for "due" :P

3. Perpetuity

(a) The PV of a perpetuity-immediate is:

$$a_{\overline{\infty}|} = \frac{1}{i}$$

(b) The PV of a perpetuity-due is:

$$\ddot{a}_{\overline{\infty}|} = \frac{1}{d}$$

(c) The PV of a perpetuity-due exceeds the PV of a perpetuity-immediate by the payment of 1 at time 0:

$$\frac{1}{d} - \frac{1}{i} = 1$$

- Calculator Requirement

  Calculators – For all exams (except EA exams): only the following Texas Instrument calculator models may be used:

  BA-35 TI-30Xa

  BAII Plus* TI-30XIIS*

  BA II Plus Professional Edition* TI-30XIIB*

  TI-30XS MultiView* TI-30XB MultiView*

- Calculator methods review+ Problems!

EXAMPLE 1

You are given:

$$A(t) = Kt^2 + Lt + M \quad \text{for } 0 \le t \le 2$$

$$A(0) = 100$$

$$A(1) = 110$$

$$A(2) = 136$$

Determine the force of interest at time $t = \frac{1}{2}$. (Answer to nearest 0.001)

(A) 0.030 (B) 0.049 (C) 0.061 (D) 0.095 (E) 0.097

3. On 1/1/97 Kelly deposits X into a bank account. The account is credited with simple interest at the rate of 10% per year.

On the same date, Tara deposits X into a different bank account. The account is credited interest using a force of interest:

$$\delta_t = \frac{2t}{(t^2 + k)}.$$

From the end of the 4th year until the end of the 8th year, both accounts earn the same dollar amount of interest.

Calculate k. [SOA 11/96 #1]

(A) 96 (B) 104 (C) 112 (D) 120 (E) 128

22. Find the PV of a 10-year annuity-immediate with annual payments of \$1,200 at an effective interest rate of 6% per annum.

The keystrokes are:

2nd [CLR TVM]

(Just a reminder to always clear the TVM registers at the start of a new problem.)

10 N 6 I/Y 1,200 PMT CPT PV

The answer is −\$8,832.10 to 2 decimals.

27. John deposits \$250 at the end of each month for 5 years in an account that credits interests at a nominal rate of 6% per annum compounded monthly. How much is in his account on the date of the last deposit?

In this problem, one of the four variables is FV, rather than PV. Since we want the accumulated value on the date of the last deposit, the answer in symbolic form is $250s_{\overline{60}|}$ at 0.5%. The keystrokes are:

60 N .5 I/Y 250 +/− PMT CPT FV

The answer is \$17,442.51.

30. Find the PV of an annuity-due with payments of \$1,800 every 6 months for 8 years at a nominal rate of interest of 5% per annum compounded semiannually.

This problem asks for the PV of an annuity with payments at the *beginning* of each 6-month period for 16 periods, at an interest rate of 2.5% per period. The answer in symbolic form is $1{,}800\ddot{a}_{\overline{16}|.025}$ or $1{,}800(1.025)a_{\overline{16}|}$ or $1{,}800(1 + a_{\overline{15}|})$.

The calculator has a [BGN] key, secondary to the PMT key, which can be used to compute an annuity-due. We will do the problem with and without using the [BGN] key.

32. Determine the AV of 13 annual deposits of \$1,429 one year after the last deposit, at 2.10% effective.

In symbolic form, the answer is $1{,}429\ddot{s}_{\overline{13}|2.10\%}$ or $1{,}429(1 + i)s_{\overline{13}|}$ or $1{,}429(s_{\overline{14}|} - 1)$.

a. Without using the [BGN] key

Make sure that you are in [END] mode. The keystrokes are:

13 N 2.10 I/Y 1,429 +/− PMT CPT FV × 1.021 =

The answer is \$21,551.03.