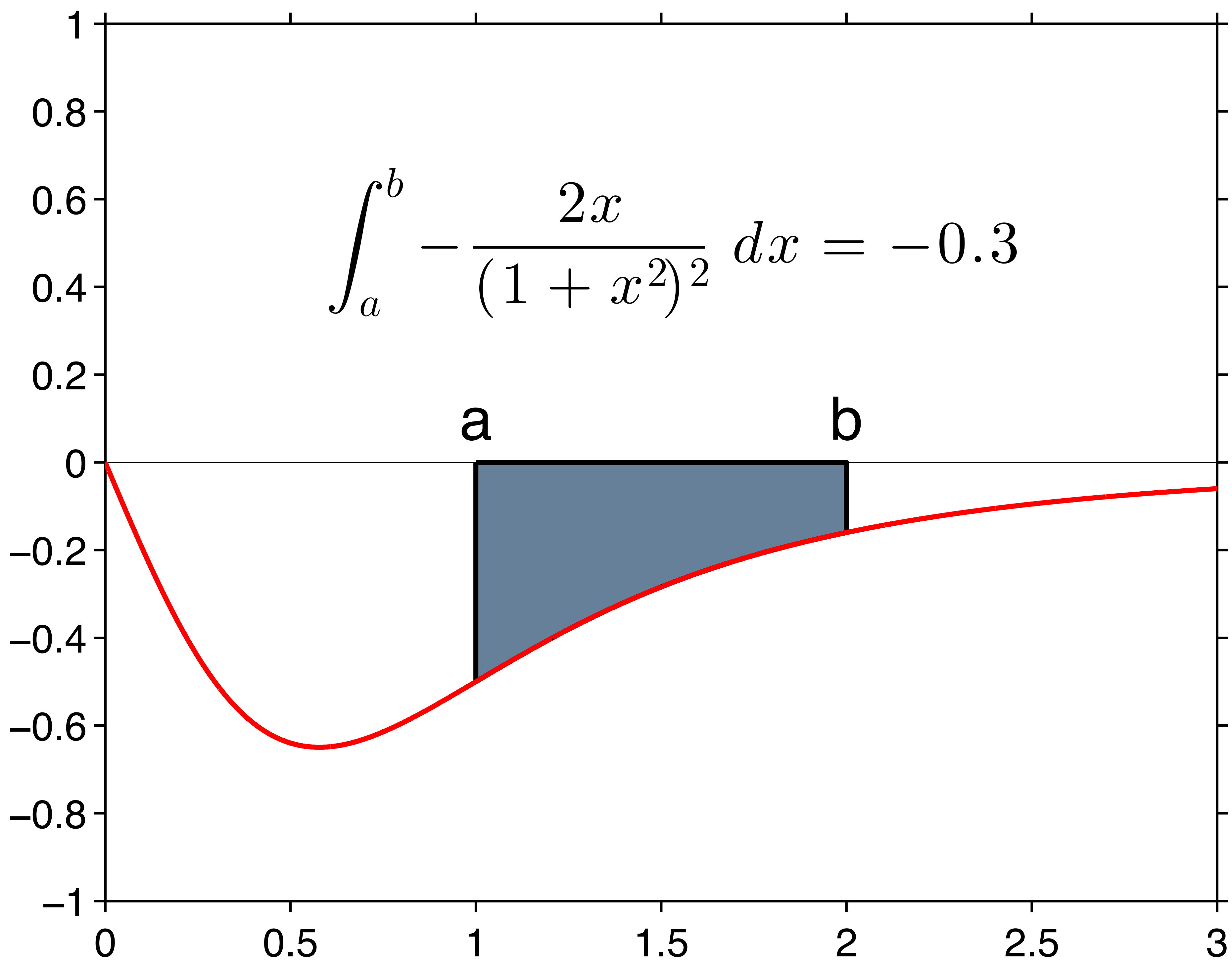
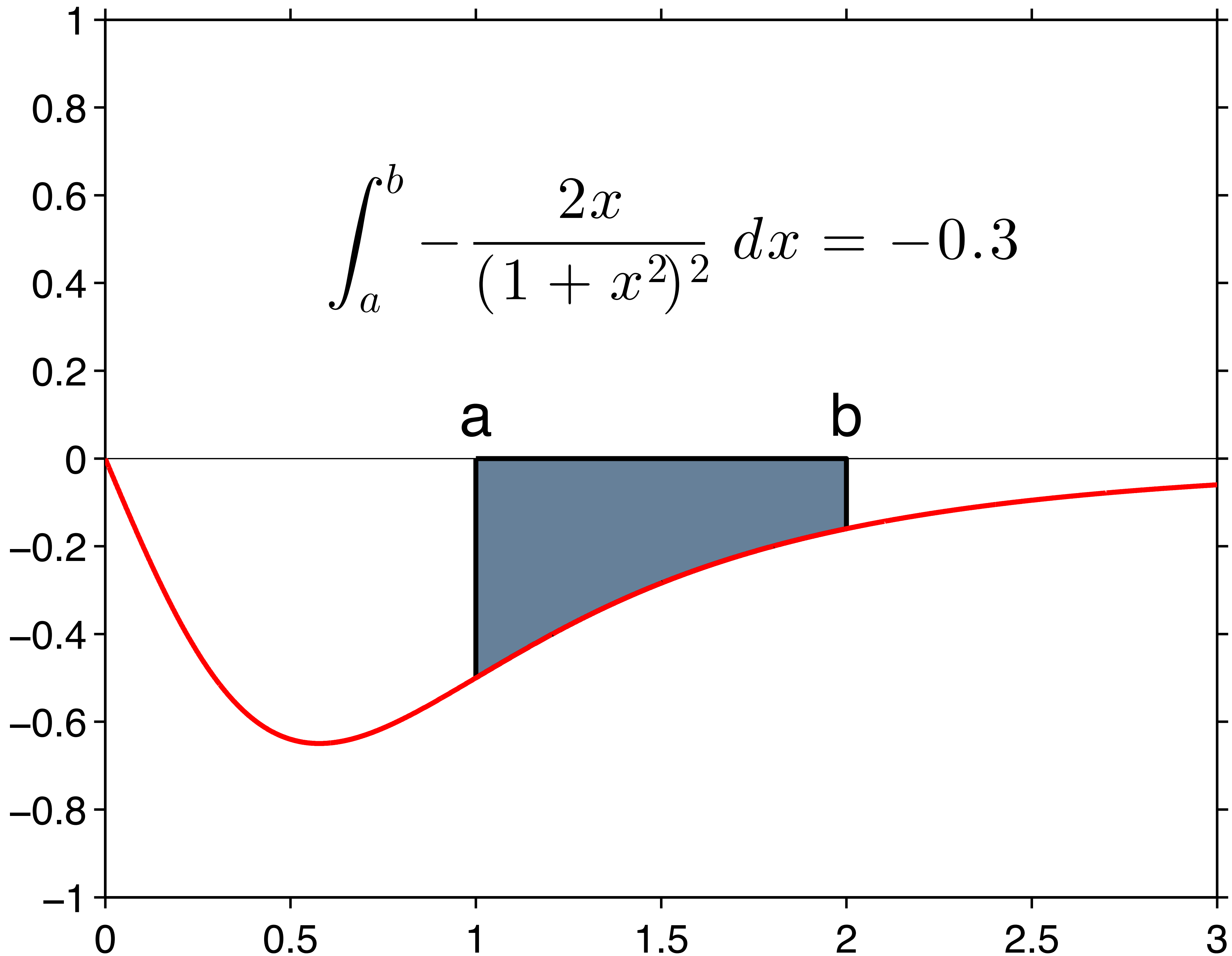
- Quadrature Challenge
- Multi-dimensional Monte Carlo Integration
- Simulating radioactive decay using random numbers
- Using random numbers to estimate π

Challenge: Create this plot



Challenge: Create this plot

Tips: Use the text and patch commands



Using Random Numbers

We saw last time:

Monte Carlo approaches are useful for:

- High dimensional quadrature.
- Physical simulation (e.g. radioactive decay).

This time we will use random numbers for:

- Estimation of π.
- Compare speed and accuracy of 3D quadrature.

3D Quadrature Example

$$Q = \int_0^1 \int_0^1 \int_0^1 x^2 y^7 z^5 \; dx\; dy\; dz$$

Monte Carlo methods use random samplings to approximate probability distributions. This technique has applications from weather prediction to quantum mechanics. One use for Monte Carlo methods is in the approximation of integrals.

This is done by choosing some number of random points over the desired interval and summing the function evaluations at these points. The area of the desired interval is then multiplied by the average function evaluation from the chosen points.

$$\int_a^b f(x)\, dx \approx \frac{(b-a)\sum_1^N f(x_n)}{N}$$

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This technique can further be implemented in multiple dimensions where the process becomes more useful. A rigorous evaluation of this technique finds the error approximately $1/\sqrt{N}$. This is not very useful in the one dimension case above, as better techniques exist, but since the error is not bounded by the number of dimensions evaluated, when many dimensional integrals are evaluated, Monte Carlo methods can become increasingly effective.

3D Quadrature Example

$$Q = \int_0^1 \int_0^1 \int_0^1 x^2 y^7 z^5 \ dx \ dy \ dz$$

Q_{num}	Q_{mc}	t_{num}	t_{mc}	t_{mc}/t_{num}	N	% error
0.00694	0.000432	1.56	0.000122	7.82E-05	10	93.8
0.00694	0.00842	1.56	0.000109	6.99E-05	100	-21.2
0.00694	0.00729	1.56	0.000374	0.000239	1.00E+03	-4.93
0.00694	0.00692	1.56	0.00149	0.000953	1.00E+04	0.304
0.00694	0.0069	1.56	0.0111	0.00713	1.00E+05	0.609
0.00694	0.00695	1.56	0.125	0.0798	1.00E+06	-0.0557
0.00694	0.00694	1.56	1.29	0.824	1.00E+07	0.0755

Challenge: Calculate π using random numbers

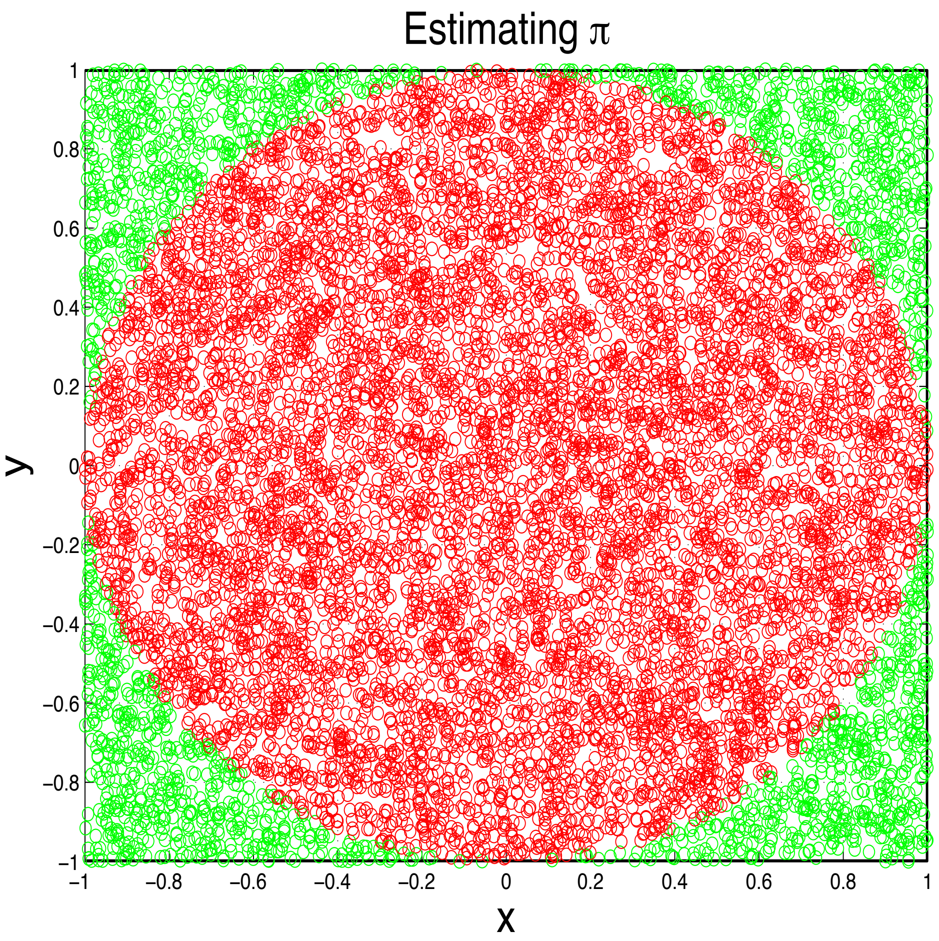
- Generate random (x,y) points in the square whose bottom left corner has coordinates (-1,-1) and whose top right corner has coordinates (1,1). This square has an **area of 4 units**.

- Circumscribed by this square is the **unit** circle of **area π units**.

- The fraction of the points that lie within the unit disk is **π/4**.

- Use these facts to calculate π.

- How does the number of random numbers generated affect the accuracy?



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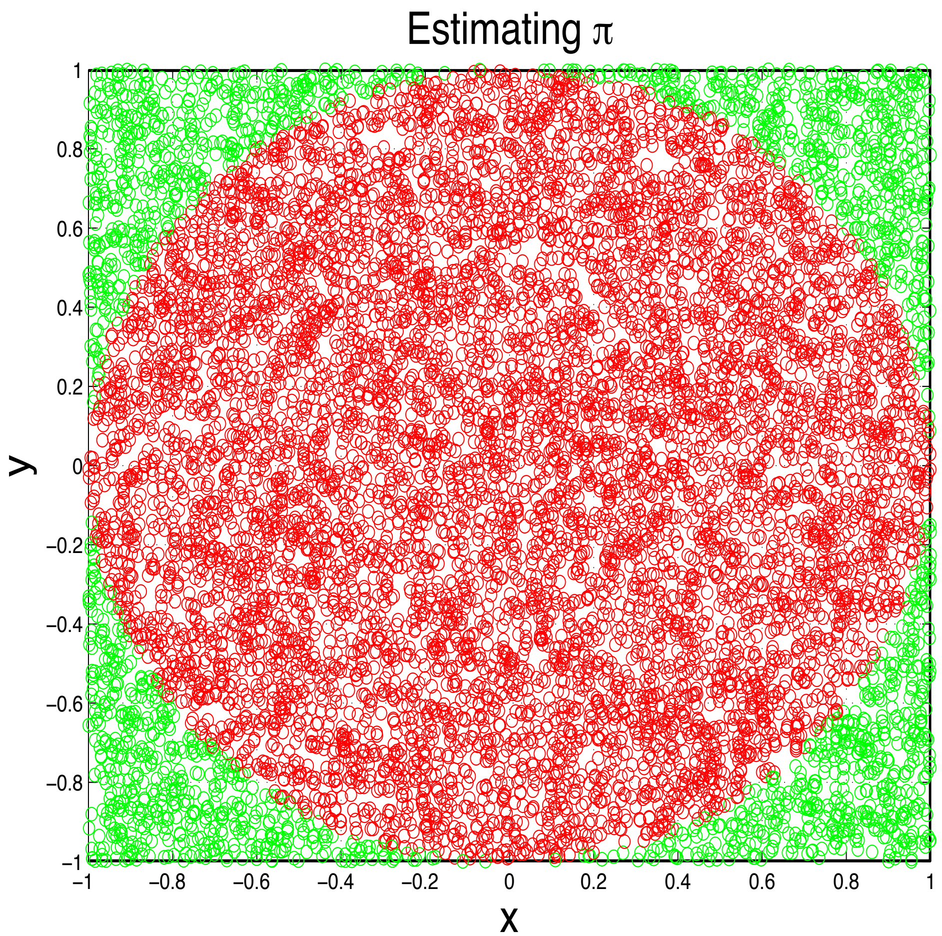

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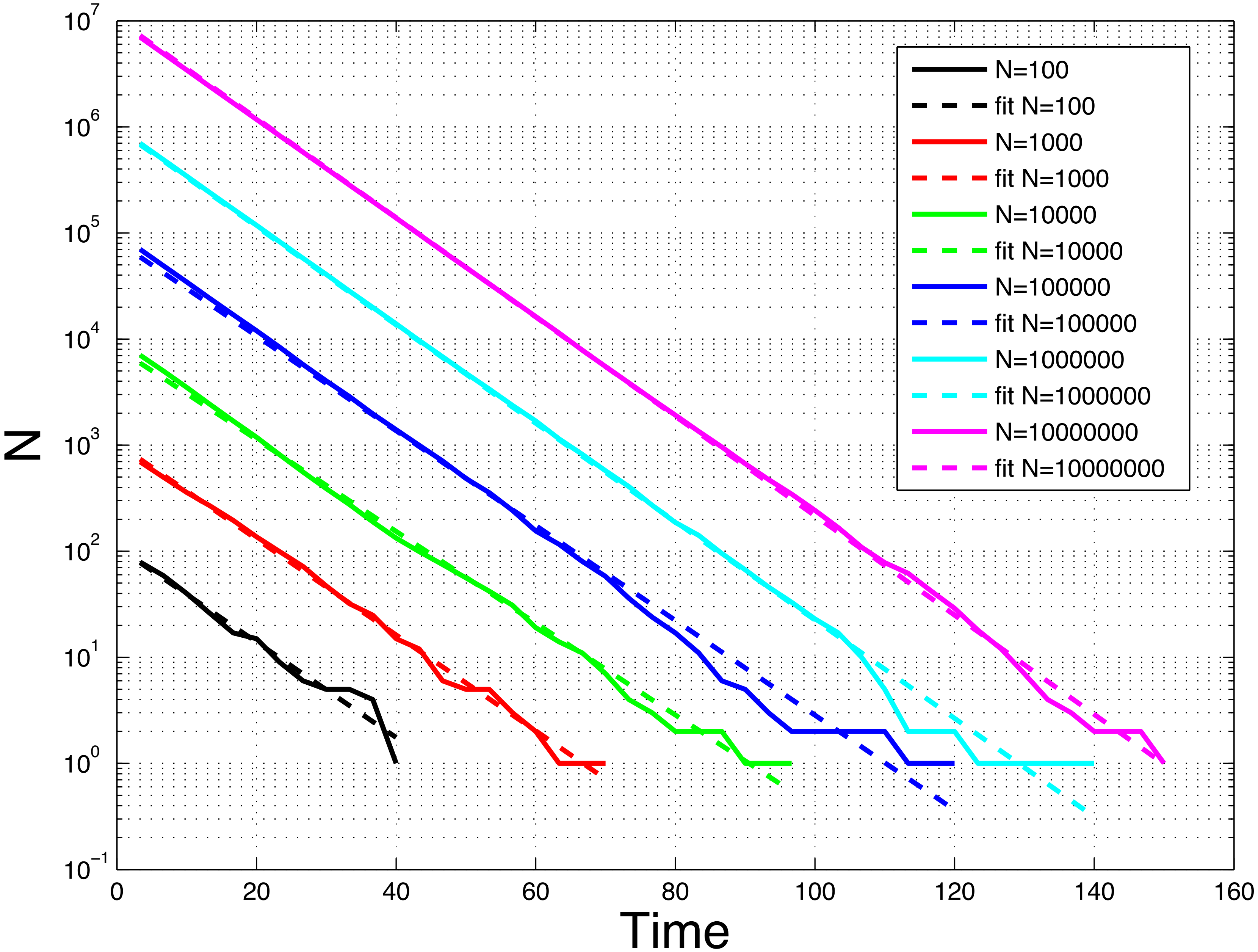
Example: Radioactive Decay

Theory: Spontaneous Decay

Spontaneous decay is a natural process in which a particle, with no external stimulation, and at one instant in time, decays into other particles.

Because the exact moment when any one particle decays is random, it does not matter how long the particle has been around or what is happening to the other particles.

Radiactive Decay, $N=N_o e^{-\lambda t}$



$$\text{decay rate} = \ln(\lambda\, \Delta N)$$

The number of decay events, -dN, over an interval dT is proportional to the number of atoms N

$$-\frac{dN}{dt} \propto N$$

The probability of decay -dN/N is proportional to dT

$$-\frac{dN}{N} = \lambda \cdot dt$$

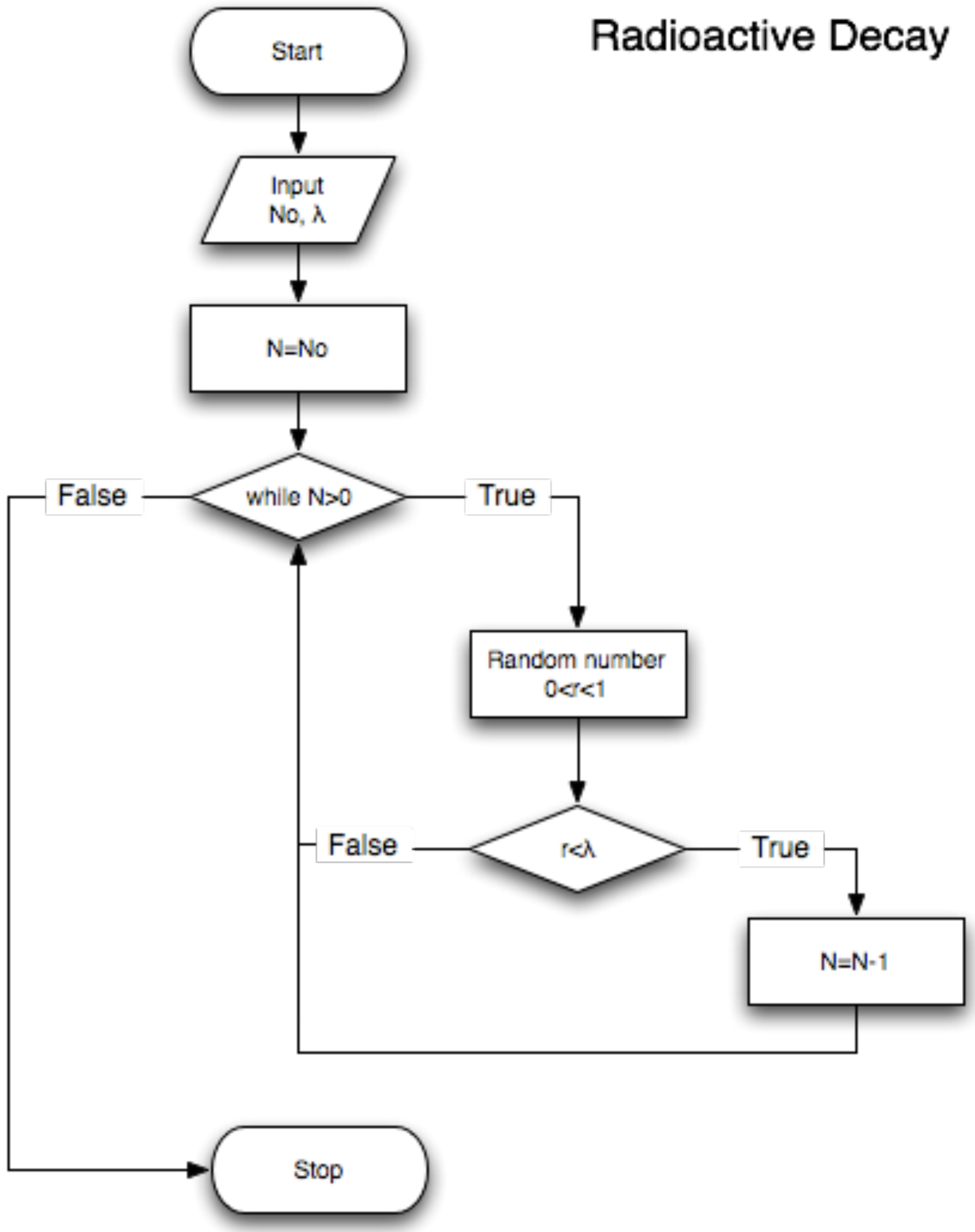
Each radionuclide has its own decay constant, λ, describing its decay. λ has units of 1/time.

$$N(t)=N_0 e^{-\lambda t}=N_0 e^{-\frac{t}{\tau}}$$

$$\tau = \frac{1}{\lambda}, \text{ and } \lambda = \frac{1}{\tau}$$

$$t_{\frac{1}{2}} = \frac{\ln 2}{\lambda} = \tau \ln 2$$

Simulating Radioactive Decay



- Implement and visualize a Monte Carlo Simulation of radioactive decay. Start with 10^n radioactive atoms where n is 2,3,4,5,6, and 7.
- First of all start with λ=0.3. Time for one generation is λ^{-1}.
- Determine when the decay starts to be stochastic.
- Plot the decay versus time.
- Plot the decay rate ($\lambda\Delta N$) versus time.
- Try a range of λ, and verify that the decay is still exponential and that λ determines the decay rate.