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Auction Problems
(Due Thursday, October 25)

1. (Sealed Bid Auction) Consider an auction for a single item. There are two bidders. Each bidder has a privately known value, drawn from a uniform distribution on [0,100]. Suppose the bidders are asked to submit sealed bids. The high bidder wins the auction, and is required to pay the average of the winning and the losing bid.

    a. Solve for the Bayesian Nash equilibrium.

    b. Compare the expected auction revenue to the two sealed bid auctions discussed in class: the first price and second price sealed bid auctions.

2. (Auction Design) A seller hopes to sell an item, and has two bidders, A and B. The seller knows that the item is either ideal for A, ideal for B or ideal for neither. The probability of each of the first two events is z, and there is a 1-2z probability the object will be ideal for neither. If the item is "ideal" for a bidder, the bidder will value winning at 100. Otherwise, a bidder will value winning at 50. The two bidders can recognize whether an item is ideal for them, and hence know their individual values.

    a. If the seller runs a 2^{nd} price auction, what is the optimal strategy for each bidder, and the Nash equilibrium?

    b. What is the expected revenue from the 2^{nd} price auction?

    c. Can the publisher raise revenue by setting a reserve price? By how much?

    d. Suppose instead that the seller announces that it will first offer to sell at a price pH, and then if no one offers to buy at that price, offer a price pL, and that if more than one bidder wants to buy at a given price, it will randomize between them in deciding the winner. Find the choices of pH and pL that maximize the seller's revenue, assuming bidders fully understand the mechanism and respond strategically.

    e. Compare the seller's revenue in (b), (c), (d), and briefly explain your results.

3. (Common Value Auctions) Consider an auction for a single item that has a common value v, where v is drawn from a uniform distribution on [0,100]. There are three bidders: A, B and C. Before the auction, bidder A receives some information: she learns whether the common value v is greater or less than 80. The other two bidders know that A is receiving this information but do not themselves learn anything more about v. The three bidders then compete in a 2^{nd} price sealed bid auction.

a. Show that A cannot do better than to bid her expected value for the item given her information, so that this is a dominant strategy for her.

b. Find the possible (symmetric pure strategy) Nash equilibrium bids for B and C, assuming that A plays her dominant strategy. What is the largest such bid?

c. What is the expected revenue from the auction, assuming B and C make the largest bid consistent with Nash equilibrium in part (b).

d. How would your analysis change if instead of learning whether or not the v was greater or less than 80, bidder A learned whether it was greater or less than 20? What then would be the expected auction revenue?

e. What would be the Nash equilibrium and expected auction revenue if A did not receive any private information before the auction?

f. What do you make of the revenue comparison between (c), (d) and (e)?

4. (Multi-Unit Auction) Suppose a seller is looking to sell four identical units of a particular good. There are five possible buyers, who have values as follows.

	Value for 1 unit	Value for 2 units
A	90	165
B	85	160
C	70	70
D	60	60
E	50	50

Note that bidders C, D and E have no interest in winning a second unit.

a. Suppose the seller runs a uniform price auction (i.e. selling to the top four bids and setting the price equal to the fifth highest bid) and the buyers truthfully bid their valuations (i.e. offering their true willingness to pay for one unit and also their true willingness to pay for two units). Who wins and what is the auction price and revenue?

b. Do any of the bidders have an incentive to bid strategically? Find a pure-strategy Nash equilibrium of the uniform price auction. What is the revenue?

c. What if the seller were to run a Vickrey auction? What would be the auction revenue? How does it compare to (a) and (b)?

d. Suppose bidder D has a value 70 for 1 unit instead of 60. How would this affect your analysis in part (a), the possible Nash equilibria in part (b), and the Vickrey auction in part (c)?