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Auction Problems II
(Due Tuesday, November 13)

1. (Assignment Auction) Suppose there are four bidders (A, B, C and D) and two items for sale (X and Y). Each bidder can win at most one item. The values are as follows:

	X	Y
A	160	100
B	130	80
C	120	70
D	60	90

   a. Find the prices that would result from a simultaneous ascending clock auction with truthful bidding, i.e. the lowest market clearing prices.

   b. Find the full set of market clearing prices.

   c. Now suppose that bidder D is not present. Find the lowest and highest market clearing prices if there is no bidder D.

   d. How does bidder D affect the (lowest and highest) market clearing prices?

2. (Room Assignment Problem) Suppose that four roommates (A, B, C, D) live in a house with four rooms (W, X, Y, Z). Their (monetary) values for the rooms are as follows:

	W	X	Y	Z
A	50	80	70	40
B	30	50	70	45
C	60	30	50	40
D	90	50	40	30

Suppose that to begin A has room W, B has room X, C has Y and D has Z. All the roommates agree that no one can be forced out of their room unless they're willing.

   a. Can we improve the efficiency of the assignment without using money? What happens if the roommates use Top Trading Cycles to re-allocate rooms.

   b. Show that the TTC allocation is not Pareto efficient if cash trades are allowed.

   c. What is the efficient assignment if cash trades are allowed?

d. Find a set of trades, involving rooms and cash that can get the roommates from the initial assignment of rooms to the assignment identified in (c), in a way that makes everyone better off.

3. (Sponsored Search Auctions) Consider an auction for three positions that will generate, respectively, 300, 200 and 100 clicks per day. There are four bidders with per-click values \$8, \$5, \$3, and \$2. Assume the seller uses a Vickrey auction to sell the positions.

   a. Solve for the Vickrey auction outcome, prices paid and seller revenue.

   b. Would the seller do better to eliminate the third position on the page and run the auction selling only two positions that generate 300 and 200 clicks?

   c. Now suppose that instead the seller runs a generalized second price auction. Find the equilibrium of the GSP auction that corresponds to the Vickrey outcome with three positions, and with just the top two positions, for sale.

   d. Supposing the seller runs a GSP auction and offers three positions for sale, can she benefit from setting a reserve price (in \$ per click)? Find the optimal reserve price, assuming that once the reserve price is set, the bidders play according to the minimum price Nash equilibrium. (Hint: you can think of the reserve price as a minimum price and consider the lowest market clearing prices given this minimum, noting that not all positions necessarily will sell.)

4. (Clock Auction with Multi-Item Demand) Consider an auction for two items (X and Y) with four bidders (A, B, and C). Each bidder potentially can win only X, only Y or X and Y. Their monetary values are as follows:

	X	Y	X and Y
A	70	50	120
B	200	200	350
C	150	170	260

   a. Explain what will happen in a simultaneous clock auction, assuming that the bidders bid straightforwardly (i.e. announce their true demands at each point in time). You can break ties to make things work our nicely.

   b. Consider B's strategy in the auction. Assuming A and C bid straightforwardly, does she have a strategy that will result in her getting a lower price?

   c. Now consider C's strategy. Is there a departure from straightforward bidding that might result in C getting a better price?