MAE 290A - Homework # 1
Numerical Methods in Science and Engineering

Prof. Alison Marsden

Due date: Thursday Oct 11th, 2012

Problem 1 - Warm up.

(a) Find the lengths and inner product of $x = (1, 4, 0, 2)$ and $y = (2, -2, 1, 3)$

(b) Find the lengths and inner product of $x = (2 - 4i, 4i)$ and $y = (2 + 4i, 4)$

(c) Decide whether the following vectors are linearly independent
$a = (1, 1, 0, 0)$, $b = (1, 0, 1, 0)$, $c = (0, 0, 1, 1)$, $d = (0, 1, 0, 1)$

Problem 2 - Matrix multiplication. Let $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix}$ and $B = \begin{pmatrix} 6 & 5 & 4 \\ 3 & 2 & 1 \end{pmatrix}$. Calculate AB and BA by hand and with Matlab.

Problem 3 - Einstein Notation.

(a) Using Einstein and index notation, show $a \times (b \times c) = (a \cdot c)b - (a \cdot b)c$. Hint: you may need to review the use of the permutation and Kroneker delta symbols.

(b) Express the product of matrices A and B (AB) using Einstein's notation, where A is a matrix of size m by p and B is a matrix of size p by n.

Write a matlab function that calculates the product of A and B. Use the **size** function to calculate the dimensions of A and B so that your program works for any m, n, and p.

Problem 4 - Matrix and Vector Norms.

(a) The **vector** p**-norm**, denoted $|\mathbf{v}|_p$, is defined, for $1 \leq p \leq \infty$, by
$|\mathbf{v}|_p = (|\mathbf{v}_1|^p + |\mathbf{v}_2|^p + ... + |\mathbf{v}_n|^p)^{1/p}$.
For $\mathbf{v}$ = (1, -2, 3), calculate
(i) the **vector 1-norm**, a.k.a. the **Manhattan Norm**
(ii) the **vector 2-norm**, a.k.a. the **Euclidian Norm** of $\mathbf{v}$.

(b) Calculate the **Frobenius norm** of the matrix: $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$

Problem 5 - Geometrical interpretation of the inner product. Show that the inner product of a pair of two or three-dimensional vectors, v and u, can be expressed as

$$v^T u = v \cdot u = |u||v| \cos\theta$$

where θ is the angle subtended between the two vectors in the two-dimensional plane or three-dimensional space. Explain in words how this concept is used in Grahm-Schmidt orthogonalization.

Problem 6 - Grahm Schmidt Orthogonalization. Without using the built-in Matlab functions, write a general short program to create a set of orthonormal vectors from a given set of N vectors of length N. Use this program to find a set of orthonormal vectors based on the columns of

$$A = \begin{bmatrix} 0.5 & 0.4 & 0.3 & 0.2 & 0.1 \\ 0.1 & 0.2 & 0.7 & 0.1 & 0.8 \\ 0.1 & 0.2 & 0.0 & 0.1 & 0.0 \\ 0.2 & 0.1 & 0.0 & 0.1 & 0.0 \\ 0.1 & 0.0 & 0.0 & 0.5 & 0.1 \end{bmatrix}$$

Put the resulting vectors into an orthogonal matrix, and verify that $Q^T = Q^{-1}$.

Problem 7 - The Givens Rotation Matrix.

(a) Compute the Givens rotation matrix G(1,2,$\pi/2$) in order to rotate an arbitrary vector $x \in \mathbb{R}^3$ by $\pi/2$ radians counterclockwise in the $x_1 - x_2$ plane. Compute $G^T x$, where $x = (3, 4, 1)^T$.

(b) Determine the angle, θ, and the corresponding Givens rotation matrix $G(1, 2, \theta)$ in order to rotate the vector $(3, 4, 1)^T$ in the $x_1 - x_2$ plane such that the resulting vector is zero in its x_2 component. Compute $G^T x$ to confirm that the rotation matrix so constructed has the desired effect.

Problem 8 - Householder reflector matrix Compute the Householder reflector matrix $H(w)$ in order to reflect an arbitrary vector $x \in \mathbb{R}^3$ through the plane normal to $w = (0, 1, 0)^T$. Compute $H^T x$, where $x = (3, 6, 2)^T$. Plot the results before and after reflection. Discuss.