MAE 290A - Homework # 2
Numerical Methods in Science and Engineering

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Due date: Thursday Oct 18th, 2012

Problem 1 - Condition Number.

(a) Use Gauss elimination **without** pivoting to solve the following system of equations using 3 significant figures. How much does the solution change using 6 significant figures?

$$0.0001x - y = -1 \quad (1)$$
$$2x - 2y = 4 \quad (2)$$

(b) Calculate the condition number for matrix A. Is the system above ill-conditioned? Hint: $cond(A) = ||A|| * ||A^{-1}||$, where $||.||$ is any consistent matrix norm.

(c) Now use Gauss elimination **with** pivoting to solve the system of equations using 3 significant figures. For what range of condition numbers will pivoting reduce errors due to rounding?

Problem 2 - Inverse of a triangular matrix.

(a) Write a function that computes the inverse of an upper triangular matrix. Test your function on the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 2 & 3 & 4 \\ 0 & 0 & 3 & 4 \\ 0 & 0 & 0 & 4 \end{bmatrix}.$$

(b) Calculate the computational cost (to leading order) for the algorithm used.

Problem 3 - Cramers rule. Consider the following system of three equations with three unknowns,

$$x + 2y + z = 3$$

$$2x - 3y - z = 1$$

$$3x - 2y = 4$$

(a) Compute the solution (by hand) using Cramers rule, and discuss the significance of your results concerning the existence and uniqueness of the solution.
Hint: Cramer's rule says that for any system $Ax = b$,

$$x_i = det(A_i)/det(A)$$

$$i = 1, ..., n$$

where A_i is the matrix formed by replacing the ith columnn of A by the column vector b.

(b) Compute the solution (by hand) using LU decomposition. Show all intermediate steps.

(c) If you need to solve $Ax = b$ for many different b's, is Cramer's rule or LU decomposition a better choice?

Problem 4 - LU decomposition

(a) Write a function that performs LU decomposition:
function [L, U] = LUDecomposition(A)

(b) Using your code, decompose matrix A:

$$A = \begin{bmatrix} 0.5 & 0.4 & 0.3 & 0.2 & 0.1 \\ 0.1 & 0.2 & 0.7 & 0.1 & 0.8 \\ 0.1 & 0.2 & 0.0 & 0.1 & 0.0 \\ 0.2 & 0.1 & 0.0 & 0.1 & 0.0 \\ 0.1 & 0.0 & 0.0 & 0.5 & 0.1 \end{bmatrix}.$$

(c) Compute the inverse of the matrix A in terms of the inverses of the matrices L and U.

Problem 5 - Thomas Algorithm

(a) Write a program that solves the tri-diagonal system $Tx = s$ using the Thomas Algorithm.

(b) Run the program to solve

$$\begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ 1 \\ 4 \end{bmatrix}$$

Problem 6 - Case Study

(a) Calling your function LUDecomposition (from Problem - 4), write a matlab code that solves the system $Ax = b$.

(b) Using your program, find the forces and reactions associated with the statically determinate truss shown in Figure 1. The forces (F) represent either tension or compression on the members of the truss. External reactions (H_2, V_2, and V_3) are the forces that characterize how the truss interacts with the supporting surface. The hinge at node 2 can transmit both horizontal and vertical forces to the surface, whereas the roller at node 3 transmits only vertical forces. It is observed that the effect of the external loading of 1000 lb is distributed among the various members of the truss. Solve for F_1, F_2, F_3, H_2, V_2, and V_3.

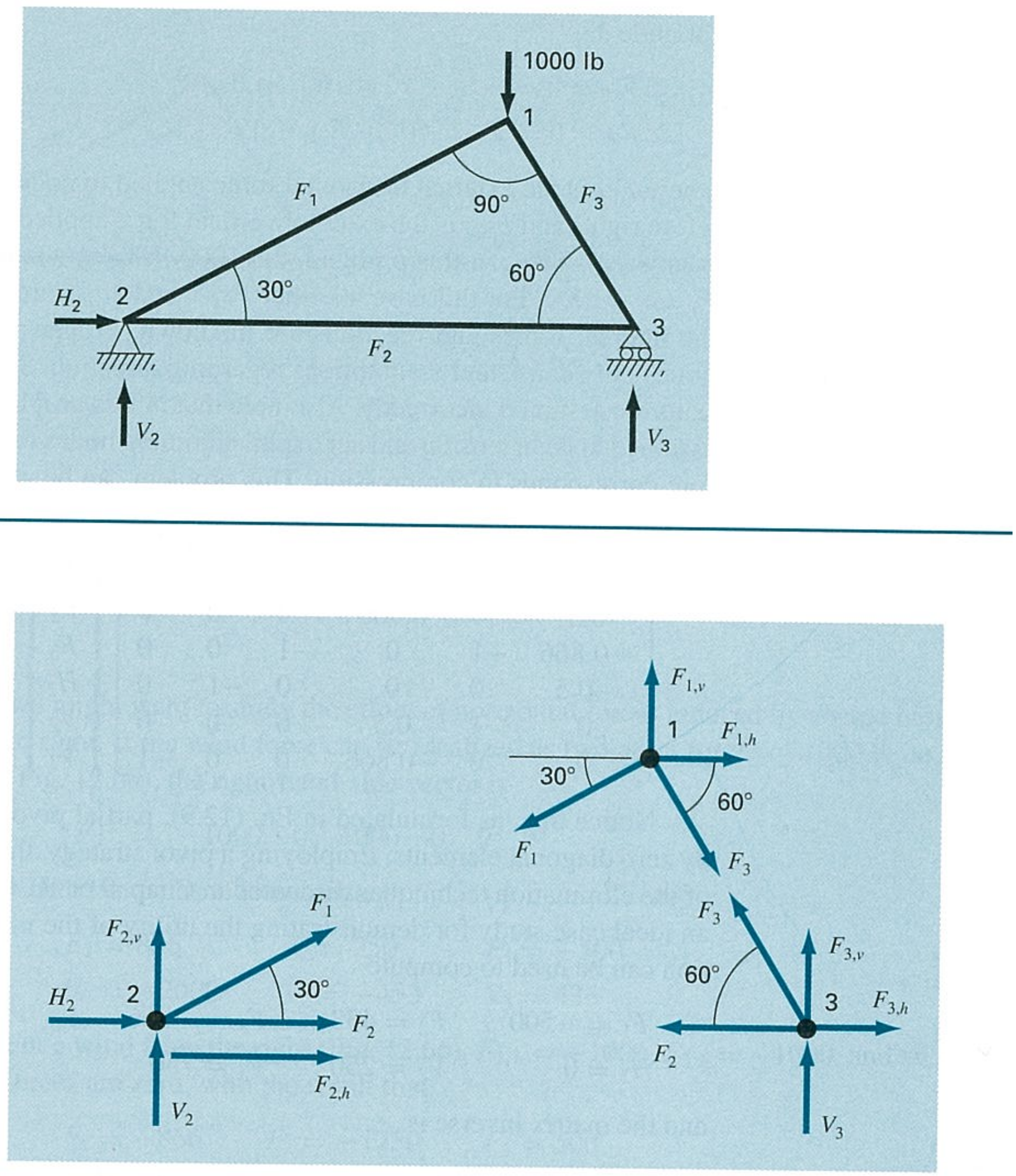

Figure 1: Problem - 6(b): Statically Determinate Truss