Name: ________________________ UCSC, Fall 2012

Sample Midterm Answer Key – Econ 133

Part I: Multiple Choice (*2 points each*)

Please circle the correct answer.

1) Assume you purchased a rental property for $100,000 and sold it one year later for $115,000 (there was no mortgage on the property). At the time of the sale, you paid $3,000 in commissions and $1000 in taxes. If you received $10,000 in rental income (all received at the end of the year), what annual rate of return did you earn?
   a) 6%
   b) 11%
   c) 21%
   d) 25%

2) Rational, risk-averse investors will always prefer portfolios _____________.
   a) located on the efficient frontier to those located on the capital market line
   b) located on the capital market line to those located on the efficient frontier
   c) at or near the minimum variance point on the efficient frontier
   d) Rational risk-averse investors prefer the risk-free asset to all other asset choices.

3) Which asset class has historically had the greatest variability of returns?
   a) Small stocks
   b) Large stocks
   c) Long-term government bonds
   d) Treasury Bills

4) The measure of risk used in the Capital Asset Pricing Model is ___________.
   a) specific risk
   b) the standard deviation of returns
   c) reinvestment risk
   d) **beta**

5) Based on the CAPM, the optimal risky portfolio is __________________.
   a) **always the same for all investors**
   b) may vary from investor to investor due to tax considerations and other investor-imposed constraints
   c) may vary from investor to investor due to degree of risk aversion
   d) will never involve short-selling

Part II: Longer Questions

Please show all calculations. If you can't find an answer, assume a solution to get full credit on a later part.

1) (*15 points*) Alex Walker in Econ 133 owns a portfolio composed of three securities. Their betas, expected returns and weights in Alex's portfolio are shown here.

Security	Beta	E(r)	Weight
A	0.80	13.6%	40%
B	1.60	?	10%
C	1.04	?	50%

a) (*2 points*) What is the beta of Alex's portfolio?

$$\beta_p = (.8 * .4) + (1.6 * .1) + (1.04 * .5) = 1$$

b) (*2 points*) Suppose that the expected return on the market portfolio is 16%, what is the expected return on Alex's portfolio?

$E(r_p) = r_F + \beta_p (E(r_m) - r_F)$. Since $\beta_p = 1, E(r_p) = E(r_m) = 16\%$

c) (*2 points*) Assuming the CAPM holds, what is the risk-free rate?

$r_f = [E(r_i) - \beta\, E(r_M)] / (1 - \beta) = [13.6\% - 0.8 \times 16\%] / (1 - 0.8) = 4\%$

d) *(4 points)* Assuming the CAPM holds, what is the expected return of the portfolio?
$E(r_B) = 4\% + 1.6\ (16\% - 4\%) = 23.2\%$
$E(r_C) = 4\% + 1.04\ (16\% - 4\%) = 16.48\%$
$E(r_P) = .40 * 13.6\% + .10 * 23.2\% + .50 * 16.48\% = 16\%$

e) *(3 points)* Given an investment budget of $10,000, how much should you invest in T-bills and how much in the portfolio to achieve an expected return of 8%? How much should you invest in security A?
$8\% = w_P\ 16\% + (1 - w_P)\ 4\% \Rightarrow w_P = .3333$

w_F=.6667	Invest .6667 x $10,000= $6,667 in T-Bills
w_p=.3333	Invest .3333 x $10,000= $3,333 in the risky portfolio, of which
	.40 x $3,333 = $1,333 in security A
	.10 x $3,333 = $333 in security B
	.50 x $3,333 = $1,667 in security C

f) (*2 points*) Since Alex can only work a part-time job during his school time and the limited income source restricts him from further diversifying his portfolio. Do you think that he will be better off investing his money in some index fund (say, S&P 500 index fund from Vanguard Group) instead of holding his current portfolio? Explain.

Alex is better off investing in an index fund, since an index fund has much lower non-systematic risk than a three-security portfolio, due to the benefits from diversification.

2) (*5 points*) The yield on Treasury notes is 4%, and the expected market risk premium is 5%. You also have access to the following data:

Regression results for Trident's common stock
Dependent variable: Monthly excess return (in decimals)

	Coefficient	t-statistic
Intercept	0.2	1.2
Excess return on S&P 500	1.3	5.6
No of observations	60	

a) (*2 points*) What is the expected return on Trident stock according to the CAPM?

E(r) = 4% + 1.3 * 5% = 10.5%

b) *(3 points)* Trident's stock return has a standard deviation of 45%, while the S&P has a standard deviation of 20%. What is the fraction of risk that is attributable to systematic factors?

$$\text{Fraction of systematic risk} = \frac{\text{Systematic variance}}{\text{Total variance}} = \frac{\beta^2\sigma_M^2}{\sigma^2} = \frac{1.3^2 x .2^2}{.45^2} = .33$$

33% of the variance (risk) is explained by systematic factors.

3) *(14 points)* The table below shows the S&P 500 for the last 5 years.

Date	**Adj Close**	**HPR**	**Deviation**	**Squared**
1/3/2011	1271.87	18.44%	15.12%	0.022867
1/4/2010	1073.87	30.03%	26.71%	0.07135
1/2/2009	825.88	-40.09%	-43.41%	0.188414
1/2/2008	1378.55	-4.15%	-7.47%	0.005574
1/3/2007	1438.24	12.36%	9.04%	0.008171
1/3/2006	1280.08	--		
Sum				0.296377

a) *(3 points)* Calculate the 5 HPRs
HPR = $\text{Close}_t/\text{Close}_{t-1}$ -1 (see table)

b) *(4 points)* What are the arithmetic average and geometric average rates of return?
r_a = 3.32%
1280.08 $(1+r_g)^5$= 1271.87
$r_g=(1271.87/1280.08)^{1/5}-1$ = -0.13%
r_g is also called the CAGR (Compound Annual Growth Rate) in this context

c) *(3 points)* What is the standard deviation of returns?
SD = $(\text{sum of squared deviation} / \text{n-1})^{1/2} = (0.296377 / 4)^{0.5}$ = 27.22%

d) *(4 points)* If the past is a good guide to the future, what level do you expect the index to reach on 1/3/2012 and on 1/3/2021? It is now 1/3/2011.
$E(\text{Index})_{1/3/2012}$= 1271.87 * $(1+r_a)$ = 1,314
$E(\text{Index})_{1/3/2021}$= 1271.87 * $(1+r_g)^{10}$ = 1,256

Some Equations (not all needed for the midterm)

APR=(periods per year) x (period rate)

EAR=(1+ period rate)$^{\text{Periods per year}}$ - 1

$$r_A = \frac{\sum_{t=1}^{T} r_t}{T}$$

$$r_G = \left[(1+r_1)\times(1+r_2)\times\ldots\times(1+r_T)\right]^{1/T} - 1$$

$$r_{CAGR} = \left(\frac{P_T}{P_0}\right)^{1/T} - 1$$

Slope of CAL = $\dfrac{E(r_P) - r_f}{\sigma_P}$

$$E(r_P) = \sum_{i=1}^{n} w_i E(r_i) \ , \quad \sum_{i=1}^{n} w_i = 1$$

$$\sigma^2 = \frac{1}{T-1}\sum_{t=1}^{T}\left[r_t - r_a\right]^2$$

$$\sigma_P^2 = (w_B\sigma_B)^2 + (w_S\sigma_S)^2 + 2(w_B\sigma_B)(w_S\sigma_S)\rho_{BS}$$

$$\sigma_P = \sqrt{\sigma_P^2}$$

$$\sigma_C = w_P\sigma_P$$

$$cov(r_i, r_j) = \tfrac{1}{T-1}\textstyle\sum_{t=1}^{T}(r_{i,t} - r_{i,a})(r_{j,t} - r_{j,a})$$

$$\rho_{BS} = \frac{\text{cov}(r_B, r_S)}{\sigma_B \times \sigma_S}$$

$$r_i - r_f = \alpha_i + \beta_i[r_M - r_f] + e_i$$

$$\beta_i = \frac{\text{cov}(r_i, r_M)}{\sigma_M^2}$$

$$\beta_P = \sum_{i=1}^{n} w_i\beta_i$$

$$\sigma_i^2 = \beta_i^2\sigma_M^2 + \sigma^2(e_i)$$

$$E(r_i) = r_f + \beta_i[E(r_M) - r_f]$$

$$V_0 = \frac{D_1 + P_1}{1+k}, \ V_0 = \frac{D_1}{k-g}, \ V_0 = \sum_{t=1}^{\infty}\frac{D_t}{(1+k)^t}$$

g = ROE × b

$$E(r) = \frac{D_1}{P_0} + g$$

$$\frac{P_0}{E_1} = \frac{1-b}{k-(ROE \times b)}$$

$$V = \sum_{t=1}^{T}\frac{\text{Coupon}}{(1+r)^t} + \frac{\text{Par Value}}{(1+r)^T}$$

$$P = \frac{\text{Coupon}}{r}\left[1 - \frac{1}{(1+r)^T}\right] + \text{Par Value} \times \frac{1}{(1+r)^T}$$

$$D = \sum_{t=1}^{T} t * w_t \ ; \ w_t = \frac{CF_t/(1+y)^t}{\text{Bond price}}; \ D_{perp.} = \frac{1+y}{y}$$

$$\frac{\Delta P}{P} = -D\left[\frac{\Delta(1+y)}{1+\text{y}}\right] \text{ and } \frac{\Delta P}{P} = -D^*\Delta y$$

$D^* = D/(1+y)$

Call Holder $\begin{cases} S_T - X, & \text{if } S_T > X \\ 0, & \text{if } S_T \le X \end{cases}$

Put Holder $\begin{cases} 0, & \text{if } S_T \ge X \\ X - S_T, & \text{if } S_T < X \end{cases}$

$$C = S_0 e^{-\delta T} N(d_1) - Xe^{-rT} N(d_2)$$

$$\Delta = \tfrac{C_u - C_d}{S_u - S_d}; \ \text{B} = \frac{1}{(1+r_f)^T} \ \frac{S_u C_d - S_d C_u}{S_u - S_d}$$

C + PV(X+D) = S + P

$$F_0 = S_0 \times (1+r_f)^T$$

Short payoff = $(F_0 - S_T)$

Long payoff = $(S_T - F_0)$