

## Midterm Exam Answer Key – Econ 133

### Part I: Multiple Choice (2 points each)

Please circle the correct answer.

- 1) Used appropriately, diversification can reduce or eliminate \_\_\_\_\_ risk.
  - a) All
  - b) Systematic
  - c) **non-systematic**
  - d) None of the above
  
- 2) Asset allocation refers to \_\_\_\_\_.
  - a) choosing which securities to hold based on their valuation
  - b) investing only in "safe" securities
  - c) **the allocation of assets into broad asset classes**
  - d) bottom-up analysis
  - e) all of the above
  
- 3) Which of the following markets represents the greatest opportunity for diversification for the US investor?
  - a) Canada (correlation with US returns = 0.60)
  - b) Japan (correlation with US returns = 0.39)
  - c) Australia (correlation with US returns = 0.43)
  - d) **Italy (correlation with US returns = 0.18)**
  
- 4) The optimal risky portfolio can be identified by finding \_\_\_\_\_.
  - a) the minimum variance point on the efficient frontier
  - b) the maximum return point on the efficient frontier
  - c) **the tangency point of the capital market line and the efficient frontier**
  - d) None of the above answers is correct
  
- 5) If the correlation between stock X and the market portfolio M is zero, which of the following statement is **not** true
  - a) The beta value of stock X is zero
  - b) The covariance between stock X and the market portfolio is zero
  - c) There is no systematic risk for stock X
  - d) **There is no non-systematic risk for stock X.**

## Part II: Longer Questions

Please show all calculations. If you can't find an answer, assume a solution to get full credit on a later part. Points correspond to the number of minutes you should spend on each part.

- 1) (14 points) Following are estimates of the expected returns and standard deviations for three assets:

Asset	Expected return (EAR)	Standard Deviation	Weight in optimal risky portfolio
T-Bills	1%		0%
Stock portfolio	8%	20%	24%
Bond portfolio	2%	8%	76%

Correlation coefficient:  $\rho_{\text{Stocks, Bonds}} = 0.2$

- a) (2 points) Compute the expected return of the optimal risky portfolio.  
 $E(r_p) = .24 \times 8\% + .76 \times 2\% = 3.44\%$
- b) (2 points) Compute the standard deviation of the optimal risky portfolio.  
 $\sigma_p^2 = .24^2 \times .2^2 + .76^2 \times .08^2 + 2 \times .24 \times .76 \times .2 \times .08 = 0.0072$   
 $\sigma_p = 8.47\%$
- c) (3 points) Given an investment budget of \$5,000, how much should you invest in each of the three assets to achieve an expected return of 3% on your complete portfolio?  
 $3\% = w_F 1\% + (1 - w_F) 3.44\%$   
 $w_F = 18\%$       Invest  $.18 \times \$5,000 = \$900$  in T-Bills  
 $w_p = 82\%$       Invest  $.82 \times \$5,000 = \$4,100$  in the optimal risky portfolio, of which  
                      $.24 \times \$4,100 = \$984$  in the stock portfolio  
                      $.76 \times \$4,100 = \$3,116$  in the bond portfolio
- d) (2 points) What is the standard deviation for this complete portfolio with  $E(r) = 3\%$ ?  
 $\sigma_c = .82 \times 8.47\% = 6.95\%$
- e) (2 points) If the T-Bills have 6 months to maturity, what is the face value (the money you get at maturity) of the purchased bills (assuming odd numbers are possible)?  
 Easy way:  $FV = \$900 * (1.01)^{1/2} = 904.49$   
 Hard way: find the monthly rate first, then calculate the FV  
 $(1 + r_{\text{monthly}})^{12} - 1 = 1\% \Leftrightarrow r_{\text{monthly}} = 0.083\%$   
 $FV = \$900 * (1.00083)^6 = 904.49$
- f) (3 points) In one sentence, how do you find the optimal risky portfolio in the Markowitz Portfolio Selection Model? In particular, what do you maximize (define!) by changing what, given which constraints?  
 We maximize the Sharpe ratio,  $(E(r_p) - r_f) / \sigma_p$ , by changing the weights  $w$  of risky assets in the portfolio, subject to the constraint that the weights add up to 1.0.

- 2) (5 points) Assume that the CAPM holds. The expected return and standard deviation of the market portfolio M are:  $E(r_M) = 10\%$ ,  $\sigma_M = 20\%$ . A risk-free asset is available and the risk-free rate,  $r_f = 2\%$ .

- a) (2 points) Shares of ABC have a beta of 2. What is the expected return on ABC according to the CAPM?

$$E(r_{ABC}) = 2\% + 2(10\% - 2\%) = 18\%$$

- b) (3 points) Assume that both the CAPM and the Single Index Model hold and that the return on a share of ABC has a standard deviation of 50%. What is the fraction of risk that is attributable to non-systematic factors?

$$\text{Fraction of systematic risk} = \frac{\text{Systematic variance}}{\text{Total variance}} = \frac{\beta_{ABC}^2 \sigma_M^2}{\sigma_{ABC}^2} = \frac{2^2 \times .2^2}{.50^2} = .64$$

Fraction of non-systematic risk =  $1 - .64 = 0.36$ . 36% of the variance (risk) is explained by non-systematic (firm-specific) factors.

- 3) (14 points) The table below shows the Apple stock price for the last 4 months.

Date	Close	Adj. Close	HPR	Deviation	Squared
10/1/2012	659.39	659.39	-1.16%	-3.99%	0.001594294
9/1/2012	667.10	667.10	0.28%	-2.56%	0.000654088
8/1/2012	665.24	665.24	9.39%	6.55%	0.004290741
7/1/2012	610.76	608.15			
Sum					0.006539122

- a) (3 points) Calculate the HPRs and enter them in the table above.

$$\text{HPR} = \text{Adj. Close}_t / \text{Adj. Close}_{t-1} - 1 \text{ (see table)}$$

- b) (4 points) What are the arithmetic average and geometric average rates of return?

$$r_a = (-1.16\% + 0.28\% + 9.39\%) / 3 = 2.84\%$$

$$608.15 (1 + r_g)^3 = 659.39$$

$$r_g = (659.39 / 608.15)^{1/3} - 1 = 2.73\%$$

- c) (3 points) Annualize the arithmetic average as an APR and an EAR. Which one is better and why?

$$\text{APR} = 2.84\% \times 12 = 34.05\%$$

$$\text{EAR} = (1 + 2.84\%)^{12} - 1 = 39.89\%$$

The EAR is always better, since it accounts for compounding.

- d) (2 points) What is the standard deviation of returns?

$$\text{SD} = (\text{sum of squared deviation} / n - 1)^{1/2} = (0.006539 / 2)^{0.5} = 5.7168\%$$

- e) (2 points) What is the annualized standard deviation?

$$\text{SD}_{\text{annual}} = \text{SD}_{\text{monthly}} \times \text{SQRT}(12) = 19.81\%$$

### Some Equations (not all needed for the midterm)

APR=(periods per year) x (period rate)

EAR=(1+ period rate)<sup>Periods per year</sup> - 1

$$r_A = \frac{\sum_{t=1}^T r_t}{T}$$

$$r_G = [(1+r_1) \times (1+r_2) \times \dots \times (1+r_T)]^{1/T} - 1$$

$$r_{CAGR} = \left(\frac{P_T}{P_0}\right)^{1/T} - 1$$

$$\text{Slope of CAL} = \frac{E(r_p) - r_f}{\sigma_p}$$

$$E(r_p) = \sum_{i=1}^n w_i E(r_i), \quad \sum_{i=1}^n w_i = 1$$

$$\sigma^2 = \frac{1}{T-1} \sum_{i=1}^T [r_i - r_a]^2$$

$$\sigma_p^2 = (w_B \sigma_B)^2 + (w_S \sigma_S)^2 + 2(w_B \sigma_B)(w_S \sigma_S) \rho_{BS}$$

$$\sigma_p = \sqrt{\sigma_p^2}$$

$$\sigma_C = w_p \sigma_p$$

$$\text{cov}(r_i, r_j) = \frac{1}{T-1} \sum_{t=1}^T (r_{i,t} - r_{i,a})(r_{j,t} - r_{j,a})$$

$$\rho_{BS} = \frac{\text{cov}(r_B, r_S)}{\sigma_B \times \sigma_S}$$

$$r_i - r_f = \alpha_i + \beta_i [r_M - r_f] + e_i$$

$$\beta_i = \frac{\text{cov}(r_i, r_M)}{\sigma_M^2}$$

$$\beta_p = \sum_{i=1}^n w_i \beta_i$$

$$\sigma_i^2 = \beta_i^2 \sigma_M^2 + \sigma^2(e_i)$$

$$E(r_i) = r_f + \beta_i [E(r_M) - r_f]$$

$$V_0 = \frac{D_1 + P_1}{1+k}, V_0 = \frac{D_1}{k-g}, V_0 = \sum_{t=1}^{\infty} \frac{D_t}{(1+k)^t}$$

$$g = \text{ROE} \times b$$

$$E(r) = \frac{D_1}{P_0} + g$$

$$\frac{P_0}{E_1} = \frac{1-b}{k - (\text{ROE} \times b)}$$

$$V = \sum_{t=1}^T \frac{\text{Coupon}}{(1+r)^t} + \frac{\text{Par Value}}{(1+r)^T}$$

$$P = \frac{\text{Coupon}}{r} \left[ 1 - \frac{1}{(1+r)^T} \right] + \text{Par Value} \times \frac{1}{(1+r)^T}$$

$$D = \sum_{t=1}^T t^* w_t; w_t = \frac{CF_t / (1+y)^t}{\text{Bond price}}; D_{\text{perp.}} = \frac{1+y}{y}$$

$$\frac{\Delta P}{P} = -D \left[ \frac{\Delta(1+y)}{1+y} \right] \text{ and } \frac{\Delta P}{P} = -D^* \Delta y$$

$$D^* = D / (1+y)$$

$$\text{Call Holder} \begin{cases} S_T - X, & \text{if } S_T > X \\ 0, & \text{if } S_T \leq X \end{cases}$$

$$\text{Put Holder} \begin{cases} 0, & \text{if } S_T \geq X \\ X - S_T, & \text{if } S_T < X \end{cases}$$

$$C = S_0 e^{-\delta T} N(d_1) - X e^{-rT} N(d_2)$$

$$\Delta = \frac{C_u - C_d}{S_u - S_d}, B = \frac{1}{(1+r_f)^T} \frac{S_u C_d - S_d C_u}{S_u - S_d}$$

$$C + \text{PV}(X+D) = S + P$$

$$F_0 = S_0 \times (1+r_f)^T$$

$$\text{Short payoff} = (F_0 - S_T)$$

$$\text{Long payoff} = (S_T - F_0)$$