Name:

Sample Final Exam – Econ 133

How to use this sample exam

To do well on the actual exam and not waste your time, you should do the following

- 1. Try the sample exam without looking at the answer key
- 2. Identify questions that you cannot answer, and consult your notes or the book to find the answer. Use the answer key only to check your answers, or if you cannot find the solution at all (bad sign)
- 3. Once you've answered all questions, put the answer key away and answer all questions again. If you get them all right without looking at other sources, you're in good shape

Part I: Multiple Choice (2 points each)

	Please circle the correct answer.
1.	Assume you purchased a rental property for \$100,000 and sold it one year later for \$115,000 (there was no mortgage on the property). At the time of the sale, you paid \$3,000 in commissions and \$1000 in taxes. If you received \$10,000 in rental income (all received at the end of the year), what annual rate of return did you earn? A) 6% B) 11% C) 21% D) 25%
2.	Rational, risk-averse investors will always prefer portfolios A) located on the efficient frontier to those located on the capital market line B) located on the capital market line to those located on the efficient frontier C) at or near the minimum variance point on the efficient frontier D) Rational risk-averse investors prefer the risk-free asset to all other asset choices.
3.	The is the stock price minus exercise price, or the payoff that could be attained by immediate exercise of an in-the-money call option. A) Intrinsic value B) Time value C) State value D) None of the above
4.	The sale of a mortgage portfolio through setting up mortgage pass-through securities is an example of A) bundling B) credit enhancement C) securitization D) unbundling

5.	Which asset class has historically had the greatest variability of returns? A) Small stocks B) Large stocks C) Long-term government bonds D) Treasury Bills			
6.	 6. If the Black-Scholes formula is solved to find the standard deviation consistent with the current market call premium, that standard deviation would be called the A) variability B) volatility C) implied volatility D) deviance 			
7.	The measure of risk used in the Capital Asset Pricing Model is A) specific risk B) the standard deviation of returns C) reinvestment risk D) beta			
8.	Based on the CAPM, the optimal risky portfolio is A) always the same for all investors B) may vary from investor to investor due to tax considerations and other investor-imposed constraints C) may vary from investor to investor due to degree of risk aversion D) will never involve short-selling			
9.	The optimal risky portfolio can be identified by finding A) the minimum variance point on the efficient frontier B) the maximum return point on the efficient frontier C) the tangency point of the capital market line and the efficient frontier D) None of the above answers is correct			
10.	Used appropriately, diversification can reduce or eliminate risk. A) all B) systematic C) non-systematic D) None of the above			

Part II: Long Questions (34 points)

Please show all calculations. If you can't find an answer, assume a solution to get full credit on a later part.

1) (14 points) Following are estimates of the expected returns and standard deviations for three assets:

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	Asset	Expected	Standard	Weight in optimal risky	
		return (EAR)	Deviation	portfolio	
	T-Bills	1%		0%	
	Stock portfolio	8%	20%	24%	
	Bond portfolio	2%	8%	76%	

Correlation coefficient: $\rho_{\text{Stocks Bonds}} = 0.2$

- a) (2 points) Compute the expected return of the optimal risky portfolio. $E(r_p)=.24 \times 8\% + .76 \times 2\% = 3.44\%$
- b) (2 points) Compute the standard deviation of the optimal risky portfolio. $\sigma_p^2 = .24^2 \text{ x } .2^2 + .76^2 \text{ x } .08^2 + 2 \text{ x } .24 \text{ x } .76 \text{ x } .2 \text{ x } .24 \text{ x } .08 = 0.0072$ $\sigma_p = 8.47\%$
- c) (3 points) Given an investment budget of \$5,000, how much should you invest in each of the three assets to achieve an expected return of 3% on your complete portfolio? 3%=w_F 1% + (1-w_F) 3.44%

$$w_F$$
=18% Invest .18 x \$5,000= \$900 in T-Bills w_p =82% Invest .82 x \$5,000= \$4,100 in the optimal risky portfolio, of which .24 x \$4,100 = \$984 in the stock portfolio

 $.24 \times \$4,100 = \984 in the stock portfolio $.76 \times \$4,100 = \$3,116$ in the bond portfolio

- d) (2 points) What is the standard deviation for this complete portfolio with E(r)=3%? σ_c =.82 x 8.47% = 6.95%
- e) (2 points) If the T-Bills have 6 months to maturity, what is the face value of the purchased bills (assuming odd numbers are possible)? $FV = \$900 * (1.01)^{1/2} = 904.49$
- f) (3 points) In general, how do you find the optimal risky portfolio in the Markowitz Portfolio Selection Model? In particular, what do you maximize (define!) by changing what, given which constraints?

We maximize the Sharpe ratio ($E(r_p) - r_f$) / σ_p by changing the weights w of risky assets in the portfolio, subject to the constraint that the weights add up to 1.0.

- 2) (12 points) Macro Systems just paid an annual dividend of \$0.32 per share. Its dividend is expected to double in each of the next four years (D_1 through D_4), after which it will grow at a more modest pace. Its expected return on equity is estimated at 10%, and Macro is expected to reinvest 10% of its profits in the business.
 - a) (2 points) If the current rate on Treasury Bills is 3% and the long-term average return on the S&P 500 is 8%, what is Macro's market capitalization rate (required return) given Macro's beta of 2.0?

$$E(r_{Macro}) = k = r_f + beta (r_m - r_f) = 3\% + 2 * 5\% = 13\%$$

- b) (2 points) What is the expected long-term growth rate? $g=ROE \times b = 10\% \times 10\% = 1\%$ per year.
- c) (3 points) What is the intrinsic value?

$$P_0 = \frac{0.64}{1.13} + \frac{1.28}{1.13^2} + \frac{2.56}{1.13^3} + \frac{5.12}{1.13^4} + \frac{5.12(1+0.01)}{0.13-0.01} \times \frac{1}{1.13^4} = \$32.91$$

d) (3 points) Macro's stock has a standard deviation of 45%, while the S&P has a standard deviation of 20%. What is the fraction of risk that is attributable to systematic factors?

Fraction of systematic risk =
$$\frac{\text{Systematic variance}}{\text{Total variance}} = \frac{\beta^2 \sigma_M^2}{\sigma^2} = \frac{2^2 x \cdot 2^2}{.45^2} = .79$$

79% of the variance (risk) is explained by systematic factors.

- e) (2 points) If Macro trades at its intrinsic value, what should be the price of a futures contract on Macro that settles in 1 year (just after the next dividend payment)? $F_0 = S_0(1+r_f)-D_1 = 32.91 (1.03) 0.64 = 33.26$
- 3) (14 points) The table below shows the Apple stock price for the last 4 months.

Date	Close	Adj. Close	HPR	Deviation	Squared
10/1/2012	659.39	659.39	-1.16%	-3.99%	0.001594294
9/1/2012	667.10	667.10	0.28%	-2.56%	0.000654088
8/1/2012	665.24	665.24	9.39%	6.55%	0.004290741
7/1/2012	610.76	608.15			
Sum					0.006539122

- a) *(3 points)* Calculate the HPRs and enter them in the table above. HPR = Adj. Close_t/Adj. Close_{t-1} -1 (see table)
- b) (4 points) What are the arithmetic average and geometric average rates of return? $r_a = (-1.16\% + 0.28\% + 9.39\%) / 3 = 2.84\%$ $608.15 (1+r_g)^3 = 659.39$ $r_g = (659.39/608.15)^{1/3} 1 = 2.73\%$
- c) (3 points) Annualize the arithmetic average as an APR and an EAR. Which one is better and why?

APR =
$$2.84\% * 12 = 34.05\%$$

EAR = $(1 + 2.84\%)^{12} - 1 = 39.89\%$

The EAR is always better, since it accounts for compounding.

- d) (2 points) What is the standard deviation of returns? SD = (sum of squared deviation / n-1) $^{1/2}$ = (0.006539 / 2) $^{0.5}$ = 5.7168%
- e) (2 points) What is the annualized standard deviation? $SD_{annual} = SD_{monthly} * SQRT(12) = 19.81\%$
- 4) (7 points) Assume that the CAPM holds. The expected return and standard deviation of the market portfolio M are: $E(r_{\rm M}) = 10\%$, $\sigma_{\rm M} = 20\%$. A risk-free asset is available and the risk-free rate, $r_f = 2\%$.
 - a) (2 points) Shares of ABC have a beta of 2. What is the expected return on ABC according to the CAPM?

$$E(r_{ABC})= 2\% + 2 (10\%-2\%) = 18\%$$

b) (3 points) Given the current dividend of \$2/share with constant growth rate 10% per year for stock ABC, what is the intrinsic value for stock ABC based on the constant-growth rate Dividend Discount (DDM) Model?
 V₀=D₁/(k-g) = 2.2/(.18-.1)=\$27.5

Since the market price is greater than the intrinsic value, the stock is overpriced.

5) (10 points) Today, Trident Inc. reported annual earnings per share (EPS) of \$4 per share, and paid out annual dividends of \$1 per share. The yield on 1-year Treasury bills is 1% (EAR), and the expected market risk premium is 5%. You also have access to the following data:

Regression results for Trident's common stock

Dependent variable: Monthly excess return (in decimals)

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	Coefficient	t-statistic
Intercept	0.2	1.2
Excess return on S&P	1.3	5.6
500		
No of observations	60	

a. (2 points) What is the expected return on Trident stock according to the CAPM?

$$E(r) = 1\% + 1.3 * 5\% = 7.5\%$$

b. (4 points) If you expect EPS to grow at 5% for 3 years, and then settle down at 2% thereafter, and a constant plowback ratio, what is the intrinsic value?

$$V_0 = \frac{1(1.05)}{1.075} + \frac{1(1.05)^2}{1.075^2} + \frac{1(1.05)^3}{1.075^3} + \frac{1(1.05)^3(1.02)}{1.075^3(.075 - .02)} = 2.86 + 17.28 = 20.14$$

c. (2 points) Where do you expect the stock price to be 2 years from now in the absence of any mispricing and if all your estimates are correct?

P grows at the same rate as dividends: $P_2 = 20.14 * (1.05)^2 = 22.21$

d. (2 points) If the stock was trading at its intrinsic value and you could borrow at the risk-free rate, what should be the futures price of a contract on Trident Inc. stock maturing in 1 year?

$$F_0 = S_0(1+r_f)^T - D = 20.14(1.01) - 1.05 = 19.29$$

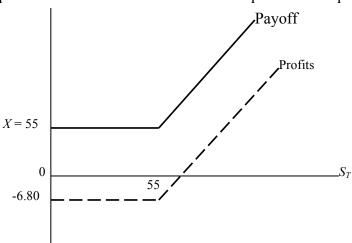
- 6) (6 points) Joe Finance has just devised the following portfolio because he is not willing to bear potential losses beyond some set level over the coming year. He buys a 1-year put option contract (100 shares) at a price (P) of \$5 and with a strike price of \$55. Joe also buys 100 shares of stock at the current price stock price of \$55. He establishes the entire portfolio with his own funds. The risk-free rate is 3%. Consider the time value of money.
 - a). (2 points) At expiration date, if the underlying stock price goes up to \$65, what are Joe's payoffs and net profit on his investment?
 - b). (2 points) At expiration date, if the underlying stock price goes down to \$52, what are Joe's payoffs and net profit on his investment?
 - c). (2 points) What type of bet is this by Joe? What is the maximum loss and maximum gain that he might have by using such a strategy?

Answers:

Initial Investment = \$5,500 in stock and \$500 in options = \$6,000 total

- (a) Payoff = value of stocks + payoff from option = \$65*100+0 = \$6,500Profit= payoff - FV(costs) = $$6,500 - $6,000 \times 1.03 = 320
- (b) Payoff = $$52*100 + (X S_T)*100 = $5,200 + ($55 $52)*100 = $5,500$ Profit = $$5,500 - $6,000 \times 1.03 = -$680$
- (c) The strategy is called a protective put. The maximum gain is unlimited, while the maximum loss is just \$680. Thus, this strategy establishes a "floor" on potential losses.

Below is a graph of the value of this "Protective Put" position at expiration (1 share + 1 put):



- 7) (*4 points*) Assume that the required rate of return (yield to maturity) on a bond is 5.2%. This bond also has 7 years until maturity and the coupon rate is 4.3%.
 - a) (2 points) What should the market price of this bond be?

Since coupon payments are made semiannually, the number of periods (T) equals 14. The coupon payments are $(0.043 / 2) \times 1000 = 1.50$. The effective discount rate (T) is also divided by 2, T = 5.2% / 2 = 2.6%. Thus,

$$P = \text{Coupon} \times \frac{1}{r} \left[1 - \frac{1}{(1+r)^T} \right] + \text{Par Value} \times \frac{1}{(1+r)^T} = 21.5 \times \frac{1}{0.026} \left[1 - \frac{1}{(1.026)^{14}} \right] + 1000 \times \frac{1}{(1.026)^{14}} = \$947.75$$

b) (2 points) What should the price be if the required rate of return increases to 6.2%? The effective discount rate (r) is now, r = 6.2% / 2 = 3.1%. Thus,

$$P = 21.5 \times \frac{1}{0.031} \left[1 - \frac{1}{(1.031)^{14}} \right] + 1000 \times \frac{1}{(1.031)^{14}} = $893.42$$

- 8) (7 points) You are evaluating the relationship between the spot price and the futures price of palladium. Palladium currently sells for \$440 per troy oz. There is a futures contract that matures in two months with a price of \$444 per troy oz. The annual risk-free rate is 3% (APR with monthly compounding).
 - a) (2 points) According to spot-futures parity, what should be the price of the futures contract?

The spot-futures parity condition implies that

$$F_0 = S_0 \times (1 + r_f)^T = \$440 \times [1 + (0.03/12)]^2 = \$442.20$$

b) (*3 points*) Does an arbitrage opportunity exist? If so, describe the actions that you could take to exploit the opportunity, and determine the accompanying cash flows.

An arbitrage opportunity exists here because the futures contract is overpriced. It is important to remember the basic arbitrage strategy for any market where you might be able to detect mispricing. In general, you should sell what is overpriced (take a short position) and buy what is relatively underpriced (take a long position). Here we should short the futures contract and buy as much palladium as we can afford. It costs nothing up front to enter the short side of the futures contract but we also do not get any immediate revenue from the futures contract. Therefore, we will have to borrow all of the money necessary to purchase palladium today at \$440 per troy oz and sell it at S_T per troy oz in two months.

The amount of palladium that we can purchase is usually constrained by how much we can borrow (our credit rating). However, for every oz that we purchase, we need to enter into an equal number of short futures positions (for the same quantity). Therefore, we can just represent the strategy in the table below for a single oz. Remember to write down how you get the profits for the short futures position (= $F_0 - S_T$)!

	(Cash Flow) ₀	Cash Flow in 2 months
Borrow S_0 and repay in 2 mos.	\$440	$-440 \times (1.0025)^2 = -\442.20
Buy Palladium now, sell in 2 mos.	-\$440	S_{T}
Enter Short Futures Position	0	$$444 - S_T$
Total Cash Flow	0	\$1.80

We will have a net cash flow of zero today and a net inflow of \$1.80 per troy oz of Palladium. This is the essential characteristic of all arbitrage strategies.

c) (2 points) If you follow your strategy from part b, what impact does the ending price of the palladium have on your profits?

The ending price of Palladium is irrelevant! The total cash flow or net profit will always equal the actual F_0 – theoretical F_0 (according to future-spot parity), which is \$444 – \$442.20 = \$1.80 in this case. The ending price of Palladium will always cancel out when calculating profits from this arbitrage opportunity.

Some Equations (not all needed for the midterm)

APR=(periods per year) x (period rate)

$$r_A = \frac{\sum_{t=1}^{T} r_t}{T}$$

$$r_G = \left[(1 + r_1) \times (1 + r_2) \times ... \times (1 + r_T) \right]^{1/T} - 1$$

$$r_{CAGR} = \left(\frac{P_T}{P_0}\right)^{1/T} - 1$$

Slope of CAL =
$$\frac{E(r_P) - r_f}{\sigma_P}$$

$$E(r_P) = \sum_{i=1}^{n} w_i E(r_i) , \quad \sum_{i=1}^{n} w_i = 1$$

$$\sigma^2 = \frac{1}{T-1} \sum_{t=1}^{T} \left[r_t - r_a \right]^2$$

$$\sigma_P^2 = (w_B \sigma_B)^2 + (w_S \sigma_S)^2 + 2(w_B \sigma_B)(w_S \sigma_S) \rho_{BS}$$

$$\sigma_P = \sqrt{\sigma_P^2}$$

$$\sigma_C = w_P \sigma_P$$

$$cov(r_i, r_j) = \frac{1}{T-1} \sum_{t=1}^{T} (r_{i,t} - r_{i,a}) (r_{j,t} - r_{j,a})$$

$$\rho_{BS} = \frac{\operatorname{cov}(r_B, r_S)}{\sigma_B \times \sigma_S}$$

$$r_i - r_f = \alpha_i + \beta_i [r_M - r_f] + e_i$$

$$\beta_i = \frac{\text{cov}(r_i, r_M)}{\sigma_M^2}$$

$$\beta_P = \sum_{i=1}^n w_i \beta_i$$

$$\sigma_i^2 = \beta_i^2 \sigma_M^2 + \sigma^2(e_i)$$

$$E(r_i) = r_f + \beta_i [E(r_M) - r_f]$$

$$V_0 = \frac{D_1 + P_1}{1 + k} \,, \ V_0 = \frac{D_1}{k - g} \,, \ V_0 = \sum_{t = 1}^{\infty} \frac{D_t}{(1 + k)^t}$$

$$g = ROE \times b$$

$$E(r) = \frac{D_1}{P_0} + g$$

$$\frac{P_0}{E_1} = \frac{1 - b}{k - (ROE \times b)}$$

$$V = \sum_{t=1}^{T} \frac{\text{Coupon}}{(1+r)^{t}} + \frac{F}{(1+r)^{T}}$$

$$P = \frac{\text{Coupon}}{r} \left[1 - \frac{1}{(1+r)^T} \right] + \frac{F}{(1+r)^T}$$

$$D = \sum_{t=1}^{T} t * w_t; \ w_t = \frac{CF_t / (1+y)^t}{\text{Bond price}}; \ D_{perp.} = \frac{1+y}{y}$$

$$\frac{\Delta P}{P} = -D \left[\frac{\Delta (1+y)}{1+y} \right] \text{ and } \frac{\Delta P}{P} = -D^* \Delta y$$

$$D^* = D/(1+y)$$

Call Holder
$$\begin{cases} S_T - X, & \text{if } S_T > X \\ 0, & \text{if } S_T \le X \end{cases}$$

Put Holder
$$\begin{cases} 0, & \text{if } S_T \ge X \\ X - S_T, & \text{if } S_T < X \end{cases}$$

$$C = S_0 e^{-\delta T} N(d_1) - X e^{-rT} N(d_2)$$

$$\Delta = \frac{c_u - c_d}{s_u - s_d}; \ B = \frac{1}{(1 + r_f)^T} \frac{S_u C_d - S_d C_u}{S_u - S_d}$$

$$C + PV(X+D) = S + P$$

$$F_0 = S_0 \times (1 + r_f)^T - D_T$$

Short payoff =
$$(F_0 - S_T)$$

Long payoff =
$$(S_T - F_\theta)$$