CS 170 Algorithms
Spring 2013 C. Papadimitriou, P. Raghavendra HW 9

Due Friday April 5th at 5pm

Instructions: Please write your name, the username for your instructional account, student ID, GSI's name and discussion section time (e.g., Wed 11am) prominently on the first page of your homework. In a separate file called `collab.txt`, list your study partners for this homework, or "none" if you had no partners.

You are welcome to form small groups (up to four people) to work through the homework, but you **must** write up all your solutions strictly by yourself, and you must acknowledge any ideas you got from others (including from books, papers, web pages, etc.). Please read the collaboration policy on the syllabus (available on Piazza).

This homework is due Friday April 5th at 5pm electronically. You need to submit it using your instructional computer account with the command "`submit hw9`". Please submit two files: `hw9.pdf` should contain your answers, and `collab.txt` should list the people you worked with, or "`none`" if you worked completely on your own.

1. **(10 pts.) All-pairs Bottleneck** There are n cities in a country. Every two cities u and v are connected by a bi-directional highway that takes $l(u,v)$ hours to drive through. A car with gas tank capacity c can only travel for c hours on a highway and has to be refueled at the cities. Give an $O(n^3)$-time dynamic programming algorithm to compute, simultaneously for all pairs (u,v) of cities, the minimum gas tank capacity needed to go from u to v.

2. **(15 pts.) Typesetting** Consider a typesetting software (such as TEX) that converts a paragraph of text into PDF documents (say). The input text is a sequence of n words of width $w_1,\ldots,w_n$. The software needs to somehow minimize the amount of extra spaces, as follows. Every line has width W, and if it contains words i through j, then amount of extra spaces on this line is $W-j+i-\sum_{i\le k\le j} w_k$, because we leave one unit of space between words. The amount of extra space on each line must be nonnegative to avoid overflowing of words. Our goal is to minimize the sum of squares of extra spaces on all lines except the last. Give an efficient dynamic programming algorithm for this problem.

3. **(30 pts.) Triangulation** Problem 6.12. It begins "You are given a convex polygon P on n vertices in the plane (specified by their x and y coordinates)..."

4. **(30 pts.) Modeling**

   (a) [**Linear Regression**] One of the most important problems in the field of *statistics* is the *linear regression problem*. Roughly speaking, this problem involves fitting a straight line to statistical data represented by points – $(x_1,y_1),(x_2,y_2),\ldots,(x_n,y_n)$ – on a graph. Denoting the line by $y=a+bx$, the objective is to choose the constants a and b to provide the "best" fit according to some criterion. The criterion usually used is the *method of least squares*, but there are other interesting criteria where linear programming can be used to solve for the optimal values of a and b. For each of the following criteria, formulate the linear programming model for this problem:

(i) Minimize the sum of the absolute deviations of the data from the line; that is,

$$\text{Minimize } \sum_{i=1}^{n} |y_i - (a + bx_i)|.$$

(*Hint:* define new variables $z_i = y_i - (a + bx_i)$, as well as non-negative variables z_i^+ and z_i^- such that $z_i = z_i^+ - z_i^-$. How to minimize $|z_i|$ by optimizing z_i^+ and z_i^-?)

(ii) Minimize the maximum absolute deviation of the data from the line; that is,

$$\text{Minimize } \max_{i=1\ldots n} |y_i - (a + bx_i)|.$$

(*Hint:* use the above hint.)

(b) [**Spaceship**] A spaceship is being designed to take astronauts to Mars and back. This ship will have three compartments, each with its own independent life support system. The key element in each of these life support systems is a small *oxidizer* unit that triggers a chemical process for producing oxygen. However, these units cannot be tested in advance, and only some succeed in triggering this chemical process. Therefore it is important to have several backup units for each system. Because of differing requirements for the three compartments, the units needed for each have somewhat different characteristics. A decision must now be made on just *how many* units to provide for each compartment, taking into account design limitations on the *total* amount of *space*, *weight* and *cost* that can be allocated to these units for the entire ship. The following table summarizes these limitations as well as the characteristics of the individual units for each compartment:

Compartment	Space (cu in.)	Weight (lb)	Cost ($)	Probability of failure
1	40	15	30,000	0.30
2	50	20	35,000	0.40
3	30	10	25,000	0.20
Limitation	500	200	400,000	

The objective is to *minimize the probability* of all units failing in all three compartments, subject to the above limitations and the further restriction that each compartment have a probability of no more than 0.05 that all its units fail.

Formulate the linear programming model for this problem. (*Hint:* Use logarithms.)

5. (15 pts.) Feasibility and Optimality. Problem 7.7. It begins "Find necessary and sufficient conditions on the reals a and b under which…"