Physics 2213a **Prof. Rick Trebino**

Practice Quiz #1a **Waves to Diffraction**

1. (40 pts) Define or describe:

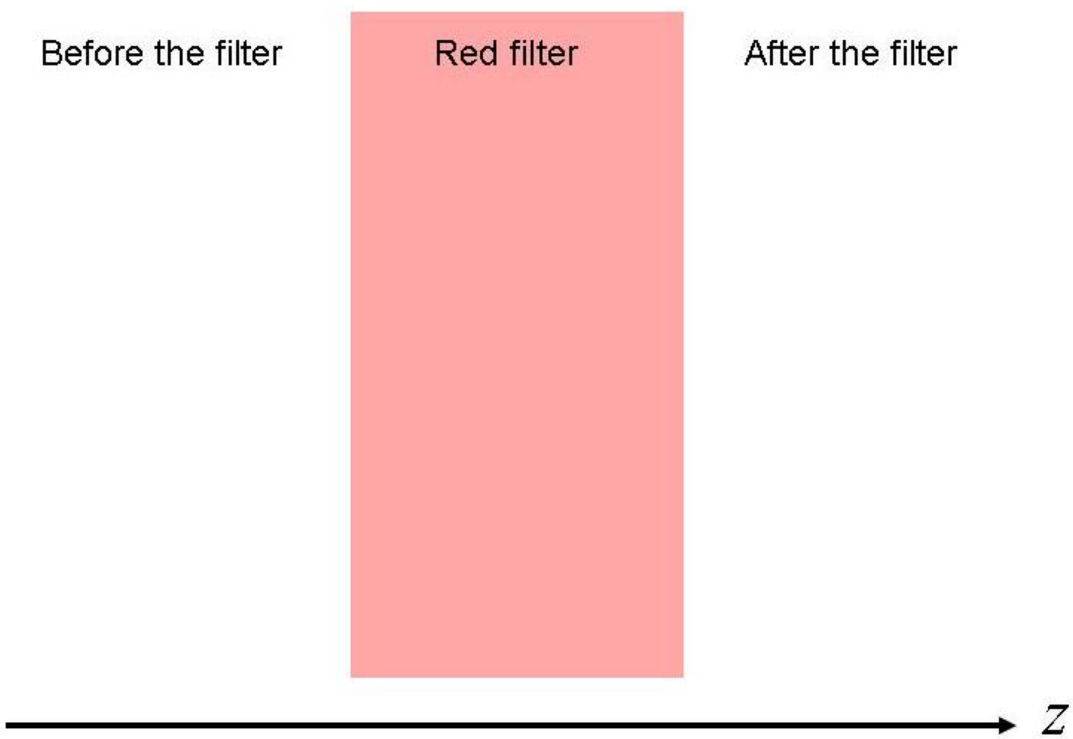
a. Diffraction

b. The wave equation

c. Electromagnetic wave (draw a picture)

d. Node

2. (20 pts) Imagine red and blue monochromatic beams of light of the form, $E(z,t) = E_0 \text{ Re}\{\exp[i(k_0 z - \omega t)]\}$, impinging on a piece of red filter glass. Plot the electric field vs. z (at a point in time) for the two beams before, inside, and after the filter.



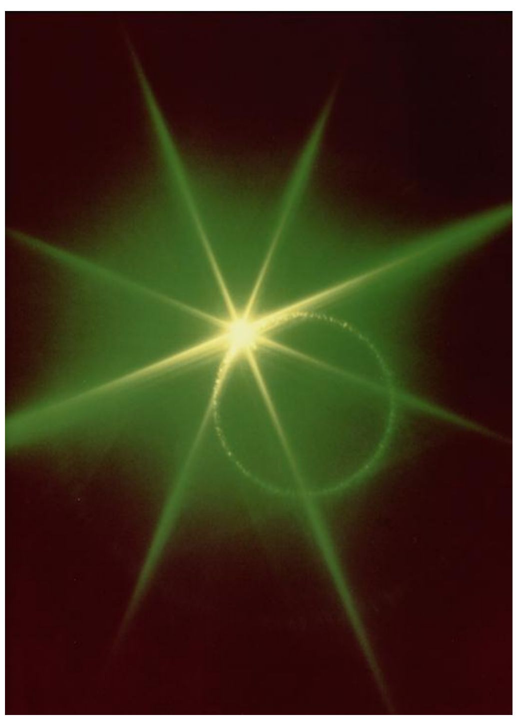
3. (20 points) The nonlinear-optical process called *second-harmonic generation* (SHG) involves taking an input beam at frequency ω and vacuum wavelength λ_0 and then generating another at the frequency 2ω and vacuum wavelength $\lambda_0/2$ in a particular medium. The condition for SHG happening is that the two beams' phase velocities are the same in the medium. Derive a simple equation for this condition. Why is this condition difficult to achieve in materials but easy in vacuum?

Answer:

If the phase velocities are the same, $c_0 / n(\lambda_0/2) = c_0 / n(\lambda_0)$, so

$$\boxed{n(\lambda_0/2) = n(\lambda_0)}$$

Alas, due to dispersion, this condition is difficult to meet. But it turns out that it can be met using cleverly designed crystals, and SHG is an impressive effect when it happens. Here's a picture I took of SHG of a 1-micron-wavelength IR beam. The green is the second harmonic (yellow is due to its saturation of the film, and the star shape is due to scattering of the green light).



4. (20 points) Describe the spectrum of a monochromatic wave in words or a plot. Now imagine a monochromatic wave passing through a shutter that opens for a time T and then closes. Write down an expression for the wave that passes through the shutter as a function of time as seen by someone just after the shutter (don't worry about the spatial dependence). And find the spectrum of the transmitted wave. Has the shutter changed the wave's spectrum?

Answer:

The spectrum of a monochromatic wave is, by definition, infinitely narrow. By the way, it's actually a Dirac delta function and so is also infinitely *high*.

The wave that passes through the shutter can be written:

$$E(t) = E_0 \, \mathrm{rect}(t/T) \exp(i\omega_0 t)$$

By the Scale Theorem,

$$\mathcal{F}\{\mathrm{rect}(t/T)\} = T \, \mathrm{sinc}(\omega T/2)$$

By the inverse Shift Theorem,

$$\mathcal{F}\{E(t)\} = \mathcal{F}\{E_0 \, \mathrm{rect}(t/T) \exp(i\omega_0 t)\} = T \, E_0 \, \mathrm{sinc}[(\omega - \omega_0)T/2]$$

And the spectrum is the squared magnitude of the Fourier transform:

$$S(\omega) = T^2 \, E_0^2 \, \mathrm{sinc}^2[(\omega - \omega_0)T/2]$$

Note that, because this sinc^2 function has a nonzero width of about $2/T$, it is broader than the original spectrum! So the shutter broadens a spectrum, and a fast enough shutter could turn, say, a green beam into a white one! This is a result of the Uncertainty Principle.