Physics 2213a, Modern Physics **Professor Rick Trebino**

Problem Set #6 **Atoms, Molecules & Lasers**

1. Show that the radial wave function R_{21} for $n = 2$ and $\ell = 1$ satisfies the Schrödinger radial equation for hydrogen. What energy results? Is this consistent with the Bohr model?

2. List all the possible quantum numbers (n, ℓ, m_ℓ) for the $n = 7$ level in atomic hydrogen.

3. Prove that the degeneracy of an atomic hydrogen state having principal quantum number n is n^2 if you ignore the spin quantum number. What is it if you include the spin quantum number?

4. Using all four quantum numbers (n, ℓ, m_ℓ, m_s), write down all possible sets of quantum numbers for the 6f state of atomic hydrogen. What is the total degeneracy?

5. Find whether the following hydrogen atom transitions are allowed, and, if they are, find the energy and wavelength involved and whether the photon is absorbed or emitted:
(a) (5, 2, 1, ½) → (5, 2, 1, -½)
(a) (4, 3, 0, ½) → (4, 2, 1, -½)
(a) (5, 2, -2, -½) → (1, 0, 0, -½)
(a) (2, 1, 1, ½) → (4, 2, 1, ½)

6. Calculate the probability of an electron in the ground state of the hydrogen atom actually being *inside* the proton (radius = 1×10^{-15} m). (Hint: note that, because the proton is so much smaller than the electron's wave function, $r \ll a_0$, you can approximate the electron's wave function as a constant over the entire proton.)

7. For all the elements through neon, list the electron descriptions of these elements in their ground state using $n\ell$ notation (for example, helium is $1s^2$).

8. Consider the NaCl molecule, for which the moment of inertia is 1.30×10^{-45} kg·m². If infrared light with a wavelength of 30 μm is incident on a gas of free NaCl molecules, what are the allowed Raman-scattered wavelengths? Hint: a wavelength of 30 μm

corresponds to a photon energy that's less than the lowest vibrational and electronic transitions, so you only need to consider rotational transitions.

9. A flashlamp pumps one third of the atoms of a two-level system into the excited state. Will it lase? If the same flashlamp pumps a three-level system with the same saturation intensity, what fraction of the atoms will be excited into level 2? Will it lase? What about a four-level system? Which of these systems will lase if the pump intensity is much larger than the saturation intensity?

10. You're now in a position to understand humankind's best prospects for designing a Star Trek-style phaser! Recall, from the second homework set, that diffraction causes a beam to broaden as it propagates a large distance, which is bad for a phaser because that means that the intensity decreases. But all materials absorb light, and they're, in fact, not simply absorbers, but "saturable absorbers." In other words, they can only absorb so much light, and then they become transparent for high intensities. If, for a two-level system in the low-intensity limit, the absorption coefficient is given by, $\alpha_0 = N_1\sigma$, where N_1 is the ground state population density and σ is the absorption cross-section (a constant), explain (in words) why a more precise expression for α that takes into account possible high intensities is $\alpha = \Delta N\sigma$, where $\Delta N = N_1 - N_2$, and N_2 is the excited-state population density. Show, using a Taylor series expansion to first order in I, that, as a result, the absorption coefficient, α, actually depends on the intensity, $I(x,y)$, of an incident beam: $\alpha(x,y) \approx \alpha_0 - \alpha_2 I(x,y)$, where α_0 is the usual absorption coefficient and α_2 is a new quantity that indicates how saturable the absorber is.

    What will be the intensity vs. x and y of a laser beam with spot size, w, after it propagates a very short distance z through a saturable absorber?

    Hint: Start with a laser beam with a Gaussian transverse electric-field profile, $\exp[-(x^2 + y^2)/w^2]$, where w is the spot size. Let its intensity on the z-axis at the entrance to the saturable absorber be I_0. Then approximate the resulting nasty-looking exponential inside an exponential: $\exp[-a\exp(-b)] \approx \exp[-a(1-b)]$. Simplify and combine the exponentials. Derive an expression for the new spot size, w', of the beam after it propagates through the medium.

This effect is called self-focusing, and it can compensate for diffraction, yielding what is often called a "light bullet." It can also cause intense laser beams to become even more intense and to damage materials.