25 Improved Boussinesq equations for surface water waves

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The main motivation of this work is the implementation of a general finite element solver for some of the improved Boussinesq models. Here, we use an extension of the model proposed by Zhao et al. (2004) to investigate the behavior of surface water waves. The equations in this model do not contain spatial derivatives with an order higher than two. Some effects like energy dissipation and wave generation by natural phenomena or external physical mechanisms are also included. As a consequence, some modified dispersion relations are derived for this extended model. A matrix-based linear stability analysis of the proposed model is presented. It is shown that this model is robust with respect to instabilities related to steep bottom gradients.

25.1 Overview

The FEniCS project, via DOLFIN, UFL and FFC, provides good technical and scientific support for the implementation of large scale industrial models based on the finite element method. Specifically, all the finite element matrices and vectors are automatically generated and assembled by DOLFIN and FFC, directly from the variational formulation of the problem which is declared using UFL. Moreover, DOLFIN provides a user friendly interface for the libraries needed to solve the finite element system of equations.

Numerical implementation of Boussinesq equations goes back to the works of Peregrine (1967) and Wu (1981), and later by the development of improved dispersion characteristics (see, e.g., Madsen et al. (1991); Nwogu (1993); Chen and Liu (1994) as well as Beji and Nadaoka (1996)).

We implement a solver for some of the Boussinesq type systems to model the evolution of surface water waves in a variable depth seabed. This type of model is used, for instance, in harbor simulation (see Figure 25.1 for an example of a standard harbor), tsunami generation and wave propagation as well as in coastal dynamics.

In Section 25.2, we begin by describing the DOLFWAVE application which is a FEniCS based application for the simulation of surface water waves (see `https://launchpad.net/dolfwave`).

The governing equations for surface water waves are presented in Section 25.3. From these equations different types of models can be derived. There are several Boussinesq models and some of the most widely used are those based on the wave surface Elevation and horizontal Velocities formulation (BEV) (see, e.g., Walkley and Berzins (2002), Woo and Liu (2004a) as well as Woo and Liu (2004b) for finite element discretizations of BEV models). However, we only consider the wave surface Elevation and velocity Potential (BEP) formulation (see, e.g., Langtangen and Pedersen (1998) for a finite element discretization of a BEP model). Thus, the number of unknowns is reduced from five (the three velocity components, the pressure and the wave surface elevation) in the BEV models to