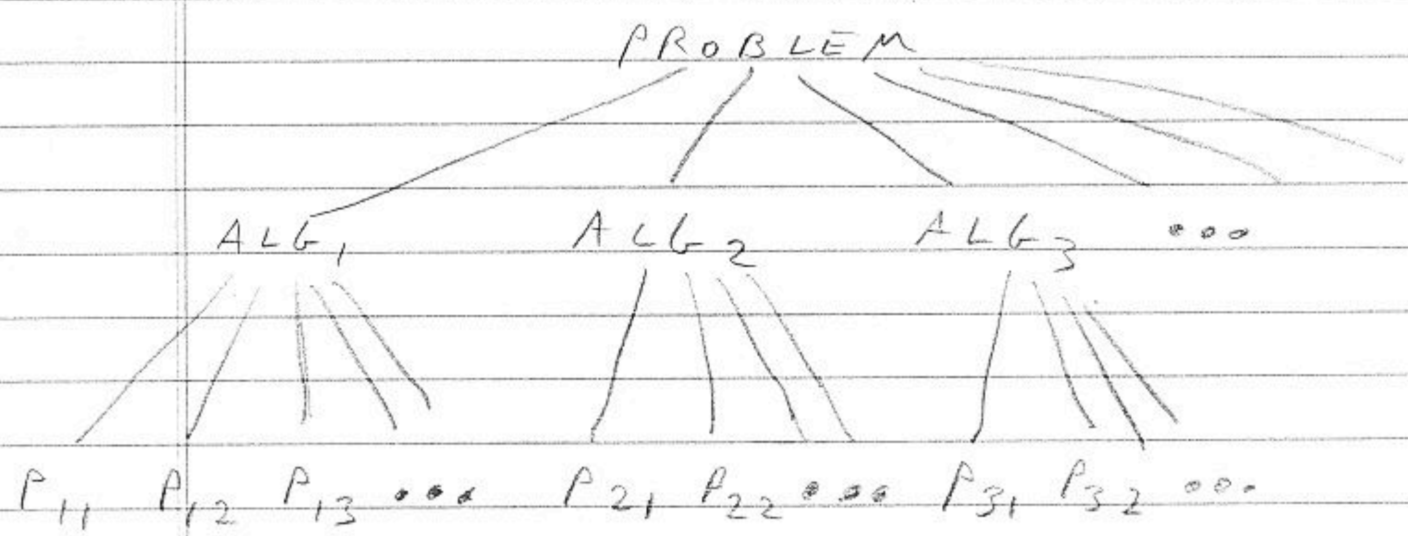
ALGORITHMS

THE BIG PICTURE



PROBLEM: INPUT → OUTPUT

ALGORITHM: STEPS TO SOLVE PROBLEM

PROGRAM: IMPLEMENTATION OF ALGORITHM

YA MĀ TĀ RĀ JA BHĀ NA SA LA GAM

0 1 1 1 0 1 0 0 0 1

0 1 1

1 1 1

1 1 0

1 0 1

0 1 0

1 0 0

0 0 0

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001
010
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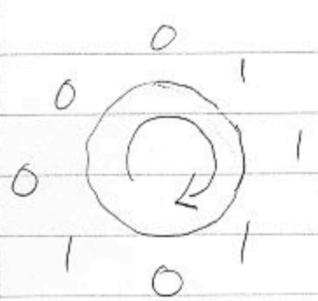
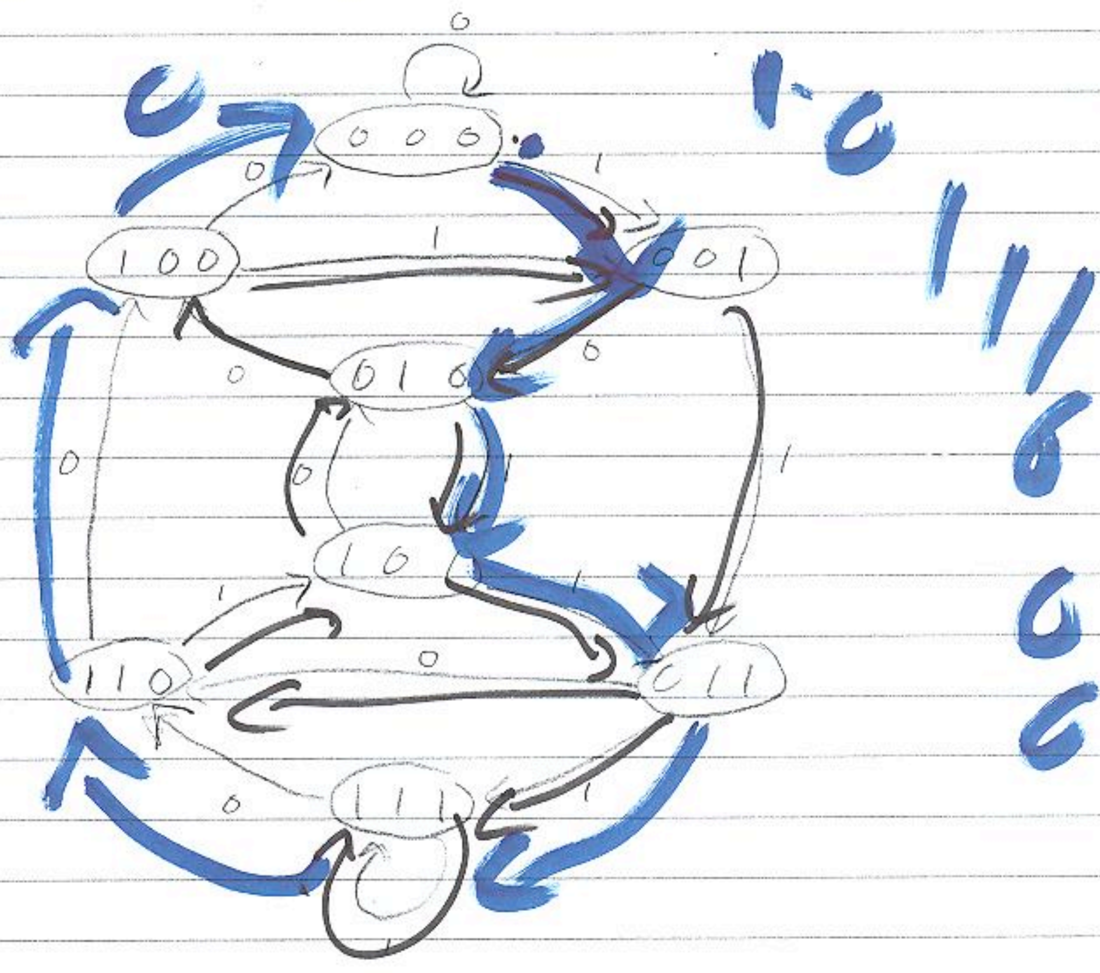


③

10011101000111

RECAST THE PROBLEM.

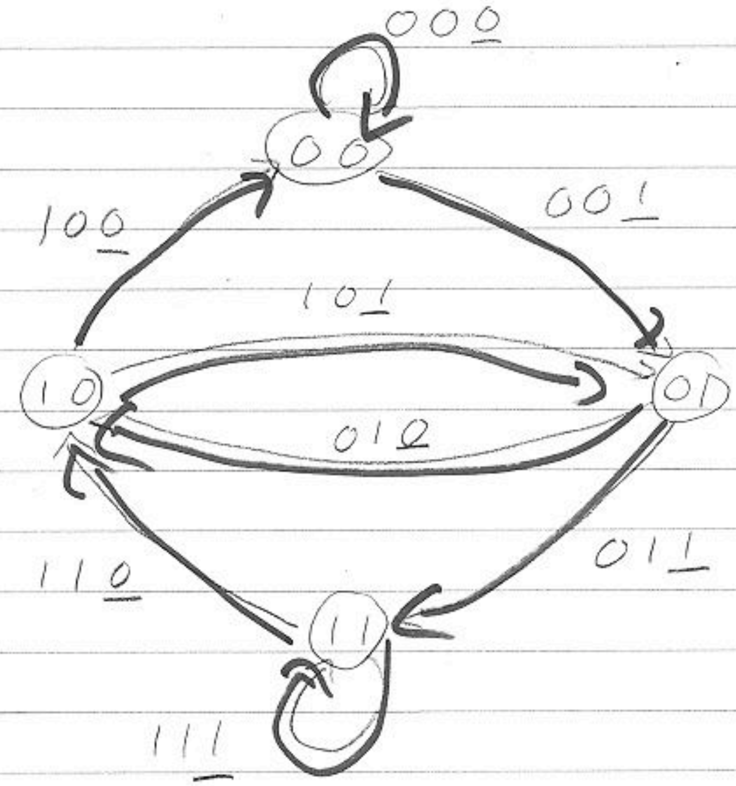


(X) ⟶ (Y) IF X OVERLAPS Y.

FIND A PATH THAT VISITS

EVERY VERTEX ONCE

RECAST ANOTHER WAY



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FIND A PATH THAT CROSSES

EVERY EDGE ONCE.

CAN GENERALIZE BEYOND BINARY

DE BRUIJN GRAPHS.

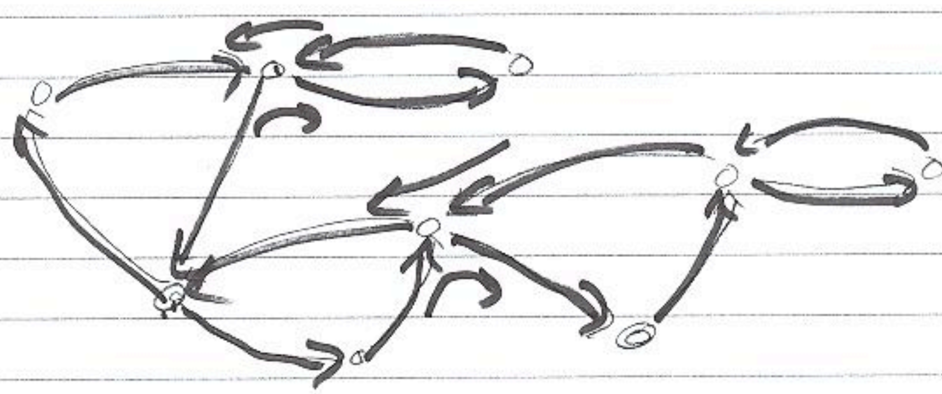
FIRST APPROACH:

FIND HAMILTONIAN CYCLE

SECOND APPROACH

FIND EULERIAN CYCLE

HOW TO FIND EULERIAN CYCLE?



[ALL DE BRUIJN GRAPHS EULERIAN]

ALGORITHM QUICK AND EASY.

HOW TO FIND HAMILTONIAN CYCLE?

THERE IS NO GENERAL ALGORITHM

THAT IS QUICK.

THE PROBLEM IS "NP-COMPLETE."

EXPECT EXPONENTIAL TIME.

WHAT CAN BE DONE?

(A) FIND APPROXIMATE SOLUTIONS

(B) HOPE YOUR DATA NOT "GENERAL"

[ACTUALLY DEBRUIJN NOT BAD.]

FIBONACCI NUMBERS

$F_0 = 0 \quad , \quad F_1 = 1 \quad , \quad F_n = F_{n-1} + F_{n-2}$

$0, 1, 1, 2, 3, 5, 8, 13, 21, 34, \cdots$

$$F_n = \left(\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right) \Big/ \sqrt{5}$$

$\sqrt{5}$ IRRATIONAL , TOO SLOW

$\phi = \dfrac{1+\sqrt{5}}{2} = 1.618 \cdots$ GOLDEN RATIO

FIBONACCI NUMBERS

COMPUTE RECURSIVELY?

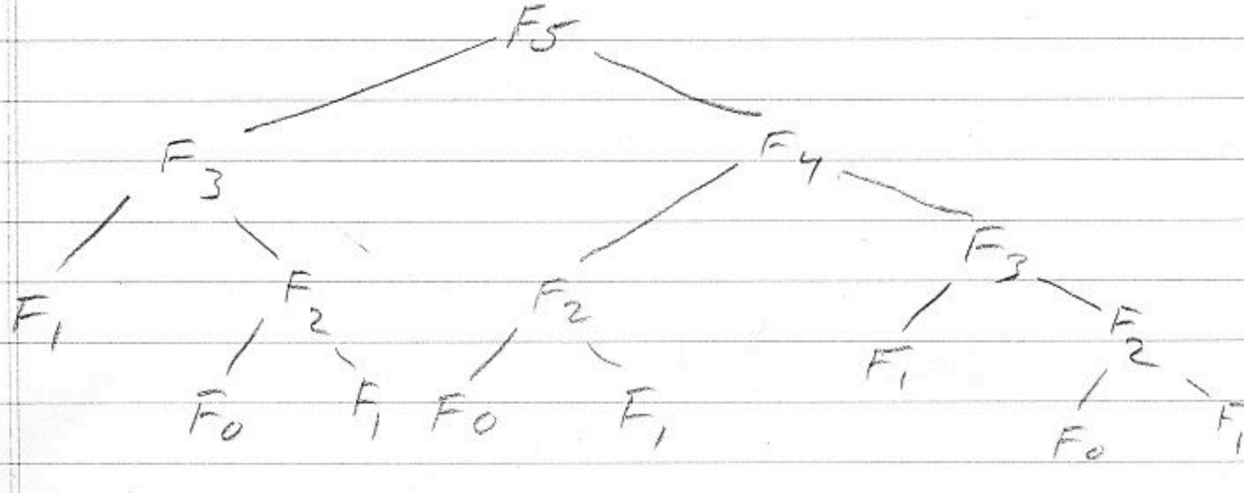
$$F_n$$

$$\Downarrow$$

$$\begin{array}{ccc} & F_n & \\ F_{n-1} & & F_{n-2} \end{array}$$

$$\Downarrow$$

$$\cdots$$



TAKES $\geq \phi^n$ STEPS!

FIBONACCI NUMBERS

USE "DYNAMIC PROGRAMMING"?

(A) REMEMBER SOLUTIONS.

(B) DO SMALL SUBPROBLEMS FIRST.

COMPUTE F_0
COMPUTE F_1
" F_2
" F_3
" F_4

⋮

COMPUTE F_n

TAKES n STEPS.

FIBONACCI NUMBERS

RECAST THE PROBLEM!

THEOREM $\begin{bmatrix} F_n & F_{n-1} \\ F_{n-1} & F_{n-2} \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}^{n-1}.$

RECALL $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \times \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} ae+bg & af+bh \\ ce+dg & cf+dh \end{bmatrix}$

PROOF BY INDUCTION.

1) TRUE FOR $n = 2$. $\begin{bmatrix} F_2 & F_1 \\ F_1 & F_0 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}^1$

2) $\begin{bmatrix} F_n & F_{n-1} \\ F_{n-1} & F_{n-2} \end{bmatrix} \times \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$

$$= \begin{bmatrix} F_n + F_{n-1} & F_n \\ F_{n-1} + F_{n-2} & F_{n-1} \end{bmatrix}$$

$$= \begin{bmatrix} F_{n+1} & F_n \\ F_n & F_{n-1} \end{bmatrix}$$

HOW TO COMPUTE X^n?

$X, X^2, X^3, X^4, X^5, \ldots$? NO

$X, X^2, X^4, X^8, X^{16}, X^{32}, \ldots$

HOW ABOUT X^{27}?

$27 = 16 + 8 + 2 + 1 = \langle 11011 \rangle_2$

SO $X^{27} = X^{16} \cdot X^8 \cdot X^2 \cdot X^1$

SINCE n IN BINARY IS $\log_2 n$ BITS.

SO X^n CAN BE COMPUTED IN

$2 \log_2 n$ MULTIPLICATIONS.

SO F_n CAN BE COMPUTED

IN $c \cdot \log_2 n$ STEPS

FOR SOME CONSTANT c.