Indian Institute of Technology Gandhinagar

PH 101-Electricity & Magnetism

Semester-II (Spring) 2012-2013

Assignment-2

Due Date**: Jan 28th, 2013, 5:30 PM**

Submit it at my office **Shed 4 -207**

Guidelines

- Write ***both*** your name and roll number **clearly** on the sheet on the top-right hand margin of the answer sheet. Also put down assignment number and date
- **Show all the necessary steps clearly and concisely**.

Problems:

1. Griffiths 2.46

Problem 2.46 The electric potential of some configuration is given by the expression

$$V(\mathbf{r}) = A\frac{e^{-\lambda r}}{r},$$

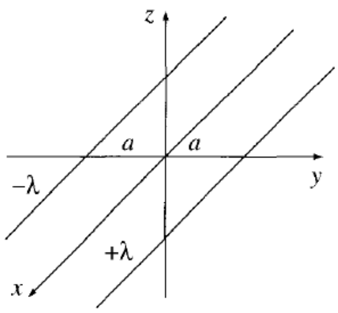
where A and λ are constants. Find the electric field $\mathbf{E}(\mathbf{r})$, the charge density $\rho(r)$, and the total charge Q. [*Answer:* $\rho = \epsilon_0 A(4\pi\delta^3(\mathbf{r}) - \lambda^2 e^{-\lambda r}/r)$]

2. Griffiths 2.47

Problem 2.47 Two infinitely long wires running parallel to the x axis carry uniform charge densities $+\lambda$ and $-\lambda$ (Fig. 2.54).

(a) Find the potential at any point (x, y, z), using the origin as your reference.

(b) Show that the equipotential surfaces are circular cylinders, and locate the axis and radius of the cylinder corresponding to a given potential V_0.



3. Griffiths 2.34: Consider two concentric spherical shells of radii *a* and *b*. Suppose the inner one carries a charge q, and the outer one charge –q (both uniformly distributed over the surface). Calculate the energy of this configuration Using
   a. $W = \frac{\epsilon_0}{2}\int_V E^2 d\tau$ (Hint: Electric field between the spheres)
   b. Expression

$$W = \frac{1}{2}\int \sigma V\, da$$

(Hint: What is the electric field outside the outer sphere? What is the potential on the inner and outer spheres?)

c. Using the expression for energy stored in a capacitor

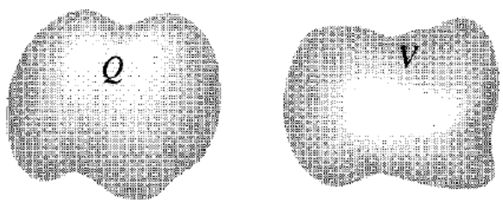
4. Griffiths 3.34 :

Problem 3.34 A point charge q of mass m is released from rest at a distance d from an infinite grounded conducting plane. How long will it take for the charge to hit the plane? [*Answer:* $(\pi d/q)\sqrt{2\pi\epsilon_0 m d}$.]

5. Griffiths Problem 3.43 :

(a) Suppose a charge distribution $\rho_1(\mathbf{r})$ produces a potential $V_1(\mathbf{r})$, and some other charge distribution $\rho_2(\mathbf{r})$ produces a potential $V_2(\mathbf{r})$. [The two situations may have nothing in common, for all I care—perhaps number 1 is a uniformly charged sphere and number 2 is a parallel-plate capacitor. Please understand that ρ_1 and ρ_2 are not present *at the same time;* we are talking about two *different problems,* one in which only ρ_1 is present, and another in which only ρ_2 is present.] Prove **Green's reciprocity theorem:**

$$\int_{\text{all space}} \rho_1 V_2 \, d\tau = \int_{\text{all space}} \rho_2 V_1 \, d\tau.$$



a *b*

[*Hint:* Evaluate $\int \mathbf{E}_1 \cdot \mathbf{E}_2 \, d\tau$ two ways, first writing $\mathbf{E}_1 = -\nabla V_1$ and using integration-by-parts to transfer the derivative to $\mathbf{E}_2$, then writing $\mathbf{E}_2 = -\nabla V_2$ and transferring the derivative to $\mathbf{E}_1$.]

(b) Suppose now that you have two separated conductors (Fig. 3.41). If you charge up conductor a by amount Q (leaving b uncharged) the resulting potential of b is, say, V_{ab}. On the other hand, if you put that same charge Q on conductor b (leaving a uncharged) the potential of a would be V_{ba}. Use Green's reciprocity theorem to show that $V_{ab} = V_{ba}$ (an astonishing result, since we assumed nothing about the shapes or placement of the conductors).

6. A sphere of radius R has a certain charge density distribution σ(θ). There are no other charges. The potential on the sphere at radius R is

$$V = V_0 + V_1 \cos\theta + V_2 \sin\theta$$

Where V_0, V_1, V_2 are constants.

a. Find $V(r,\theta)$ for $r < R$ and $r > R$

b. Find the electric field $\mathbf{E} = E_r\hat{\mathbf{r}} + E_\theta\hat{\boldsymbol{\theta}} + E_\phi\hat{\boldsymbol{\phi}}$ for $r < R$ and $r > R$ and then verify the tangential jump condition across the sphere

c. Using the normal jump condition for **E** , find $\sigma(\theta)$