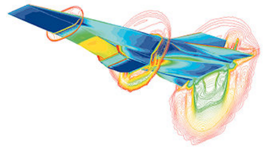
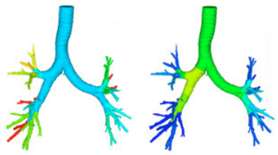
Hyper-X at Mach 7, NASA image in the public domain



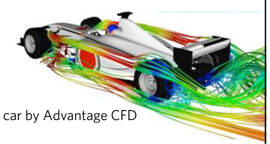
ME 702

Computational Fluid Dynamics, CFD

Prof. Lorena A. Barba



Human airways, by FuiDA nv



F1 car by Advantage CFD

BU

Discussion **Question**

- How can you recognize a conservation law?

But what is a conservation law?
It is altogether very simple in its basic logic, but can become complicated by its internal content. Conservation means that the variation of a conserved (intensive) flow quantity within a given volume is due to the net effect of some internal sources and of the amount of that quantity which is crossing the boundary surface. This amount is called the ***flux*** and its expression results from the mechanical and thermodynamic

Simplified model equations

- Recall the Navier-Stokes equation

$$\frac{\partial \vec{V}}{\partial t} + \vec{V}\cdot\nabla\vec{V} = -\frac{\nabla p}{\rho} + \nu\nabla^2\vec{V}$$

- analyze the mathematical properties of PDEs
  - how to recognize if a model describes a convection or diffusion phenomenon?
  - what other physical situations can occur?
  - what are the associated BCs and ICs?
- numerical discretization must identify these differences
  - schemes compatible with the physics

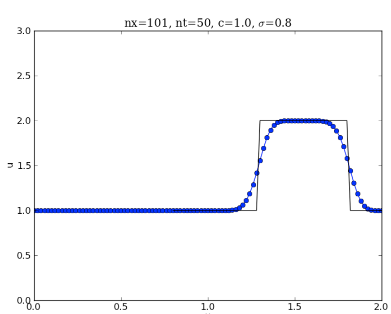
Practical module:

"The 12 steps to computing Navier-Stokes"

① — 1D **linear** convection $$\frac{\partial u}{\partial t} + c\frac{\partial u}{\partial x} = 0$$

Try different values of the parameters, to obtain various values of $\sigma = c\frac{\Delta t}{\Delta x}$

What happens?



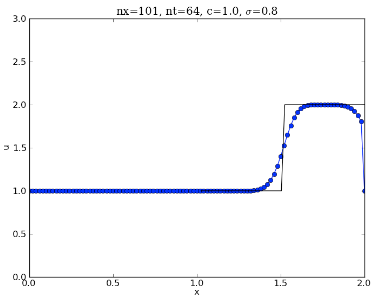
Pseudocode Step 1

```
nx = 20, nt = 50
dt = 0.01, c = 1
dx = 2/(nx-1)
```

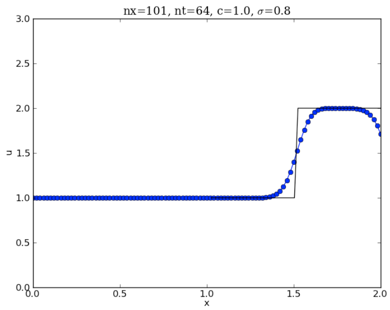
```
for i = 1:nx
 if 0.5 <= x(i) <= 1
  u(i)= 2
 else
  u(i)= 1
 end
end
```

```
for it = 1:nt
 un=u
 for i = 2:nx-1
  u(i) = un(i)- c*dt/dx*...
        ( un(i)- un(i-1) )
 end
end
```

- **Note:** we don't need to impose a BC on the right



Here, the right BC is forced at $x(2) = 2$



Here, we allow the BC to be updated in the loop over x_i

② — 1D convection

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = 0$$

Pseudocode Step 2

```
nx = 20, nt = 50
dt = 0.01
dx = 2/(nx-1)

for i = 1:nx
 if 0.5 <= x(i) <= 1
  u(i)= 2
 else
  u(i)= 1
 end
end

for it = 1:nt
 un=u
 for i = 2:nx-1
  u(i) = un(i)- un(i)*dt/dx*..
        ( un(i)- un(i-1) )
 end
end
```