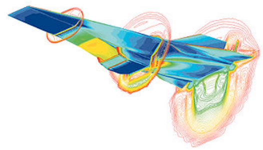
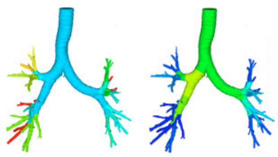
Hyper-X at Mach 7, NASA image in the public domain

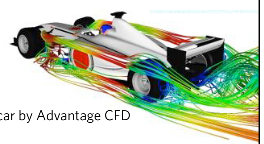


ME 702
Computational Fluid Dynamics, CFD

Prof. Lorena A. Barba



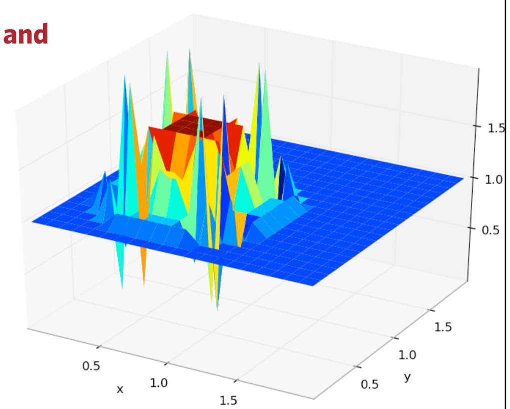
Human airways, by FuiDA nv



F1 car by Advantage CFD

BU

Consistency, Stability, and error analysis



- Recall step ① — 1D linear convection $\dfrac{\partial u}{\partial t} + c\dfrac{\partial u}{\partial x} = 0$
  - FD in time & BD in space:

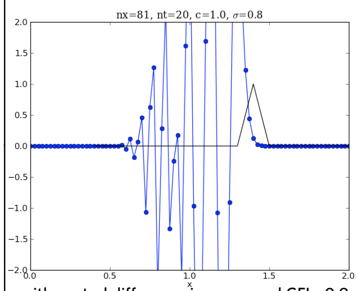
$$\frac{u_i^{n+1} - u_i^n}{\Delta t} + \frac{c}{\Delta x}\left(u_i^n - u_{i-1}^n\right) = 0$$

  - what if you used an implicit scheme?
  - what if you used a CD spatial scheme? or its implicit version?
  - what if you used FD in space? or its implicit version?

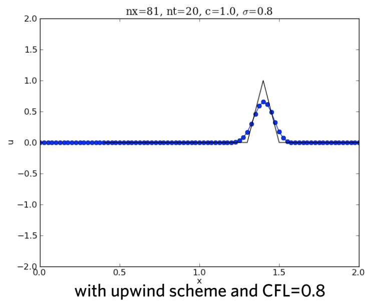
- there's unlimited choice of schemes!

- Example: $\dfrac{\partial u}{\partial t} + c\dfrac{\partial u}{\partial x} = 0$ with

$$\begin{aligned} u_0(x) &= 0 & x \le 0.9 \\ &= 10(x-0.9) & 0.9 \le x \le 1.0 \\ &= 10(1.1-x) & 1.0 \le x \le 1.1 \\ &= 0 & x \ge 1.1 \end{aligned}$$



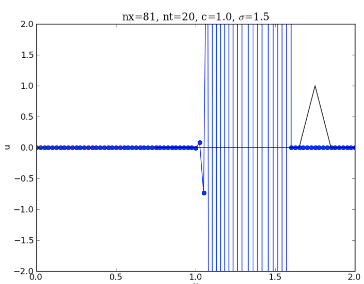
with central difference in space and CFL=0.8



with upwind scheme and CFL=0.8

- Example: $\dfrac{\partial u}{\partial t} + c\dfrac{\partial u}{\partial x} = 0$ with

$$\begin{aligned} u_0(x) &= 0 & x \le 0.9 \\ &= 10(x-0.9) & 0.9 \le x \le 1.0 \\ &= 10(1.1-x) & 1.0 \le x \le 1.1 \\ &= 0 & x \ge 1.1 \end{aligned}$$



with upwind scheme and CFL=1.5, it blows up! $\Rightarrow$ conditional stability

- Basic questions
  - What **conditions** should we impose on a numerical scheme to obtain an acceptable approximation to the problem?
  - Why do various schemes have such different behavior?
  - How can we predict their **stability**?
  - For a stable scheme, how can we obtain information on the **accuracy**?

- DEF— **Consistency**

  A condition on the numerical scheme, namely that the scheme must tend to the differential equation when the steps in time and space tend to zero.

- DEF— **Stability**

  A condition on the numerical solution, namely that all errors must remain bounded when the iteration process advances.

  That is, for finite values of the steps in time and space, the error has to remain bounded, when the number of steps tends to infinity.

- DEF— **Convergence**

  A condition on the numerical solution, that it should tend to the exact solution of the mathematical model, when time and space steps tend to zero (i.e., when the mesh is refined).