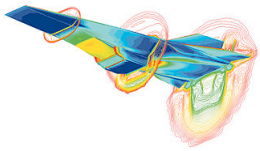
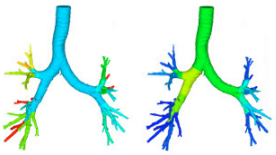
Hyper-X at Mach 7, NASA image in the public domain



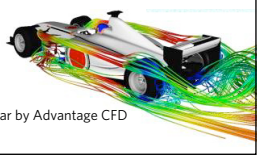
# ME 702
# Computational Fluid Dynamics, CFD

**Prof. Lorena A. Barba**





Human airways, by FuiDA nv



F1 car by Advantage CFD

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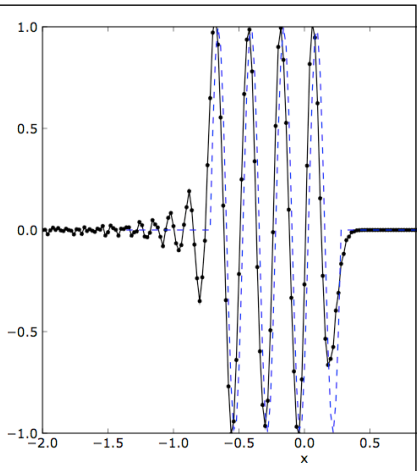
## Schemes for convection

- "Step 1" of the programming assignment used a 1st-order upwind scheme—we find it suffers from large errors (numerical diffusion)
- The central scheme (forward-time/ centered-space, or FTCS), on the other hand, is unstable and thus useless
- Other schemes have been proposed to try to improve on this!




## Lax-Friedrichs scheme

- 2nd-order central difference in space, but replace $u_i$ on the RHS by the average: $\frac{1}{2}(u_{i+1} + u_{i-1})$

$$u_i^{n+1} = \frac{1}{2}(u_{i+1}^n + u_{i-1}^n) - \frac{c\Delta t}{2\Delta x}\left(u_{i+1}^n - u_{i-1}^n\right)$$

*Draw the stencil!*

## Lax-Wendroff scheme

- Obtain 2nd-order in time by keeping the second time derivative of a Taylor expansion, then replace time derivatives by spatial derivatives. Discretize space derivatives with central difference.

$$u_i^{n+1} = u_i^n - \frac{\sigma}{2}(u_{i+1}^n - u_{i-1}^n) + \frac{\sigma^2}{2}\left(u_{i+1}^n - 2u_i^n + u_{i-1}^n\right)$$

*Draw the stencil!*



## Leapfrog scheme

- 2nd-order central difference in both space and time:

$$\frac{u_i^{n+1} - u_i^{n-1}}{2\Delta t} + \frac{c}{2\Delta x}\left(u_{i+1}^n - u_{i-1}^n\right) = 0$$

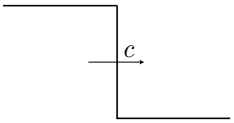
*Draw the stencil!*



## Test #1

- 1D advection: $u_t + c\,u_x = 0$
- Initial condition—Heaviside function from 1 to 0

- simulates a shock wave




- Try it first with a 1st-order upwind scheme ...

# Test #2

- A traveling sinusoidal wave packet, with two different frequencies.

- 2 periods over a distance =1

- now double the frequency