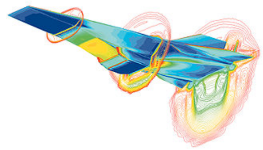
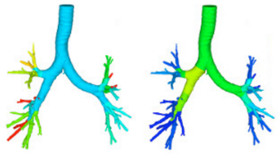
Hyper-X at Mach 7, NASA
image in the public domain

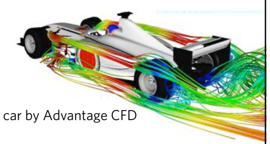


ME 702
Computational Fluid Dynamics, CFD

Prof. Lorena A. Barba



Human airways, by
FuiDA nv

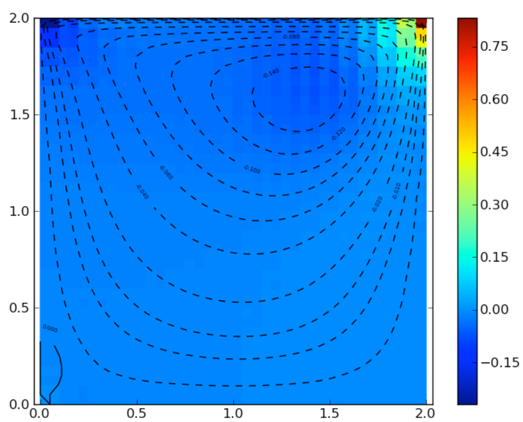


F1 car by Advantage CFD

BU

Navier-Stokes **Cavity Flow**

- Run your code with:
  - a 41 x 41 mesh, dx = dy = 0.05
  - viscosity = 0.01 (giving Re=200)



Navier-Stokes **What is happening?**

- Try this ... suppose you discretize with central difference the 1D system

$$\frac{\partial u}{\partial x} = 0$$

$$\frac{\partial u}{\partial t} + \frac{1}{2}\frac{\partial u^2}{\partial x} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\frac{\partial^2 u}{\partial x^2}$$

- Consider a “pressure correction” approach in a fractional step:
  - neglect pressure gradient from momentum equation to get temp velocity $u^\star$
  - obtain updated velocity u^{n+1} with pressure gradient
  - use this expression in the discretized continuity equation, rewrite to obtain a Poisson equation for pressure

Navier-Stokes **Discretized 1D Equations**

- continuity

$$\frac{u_{i+1}^{n+1}-u_{i-1}^{n+1}}{2\Delta x}=0$$

- momentum

$$\frac{u_i^{n+1}-u_i^n}{\Delta t}+\frac{1}{2}\frac{(u_{i+1}^n)^2-(u_{i-1}^n)^2}{2\Delta x}=-\frac{1}{\rho}\frac{p_{i+1}-p_{i-1}}{2\Delta 2}+\nu\frac{u_{i+1}^n-2u_i^n+u_{i-1}^n}{\Delta x^2}$$

- pressure correction:
  - step 1

$$\frac{u_i^\star-u_i^n}{\Delta t}+\frac{1}{2}\frac{(u_{i+1}^n)^2-(u_{i-1}^n)^2}{2\Delta x}=\nu\frac{u_{i+1}^n-2u_i^n+u_{i-1}^n}{\Delta x^2}$$

  - step 2

$$\frac{u_i^{n+1}-u_i^\star}{\Delta t}=-\frac{1}{\rho}\frac{p_{i+1}-p_{i-1}}{2\Delta x}$$

Navier-Stokes **Pressure Equation**

- step 2 gives:

$$u_i^{n+1}=-\frac{\Delta t}{\rho}\frac{p_{i+1}-p_{i-1}}{2\Delta x}+u_i^\star$$

- use this expression in the discretized continuity equation:

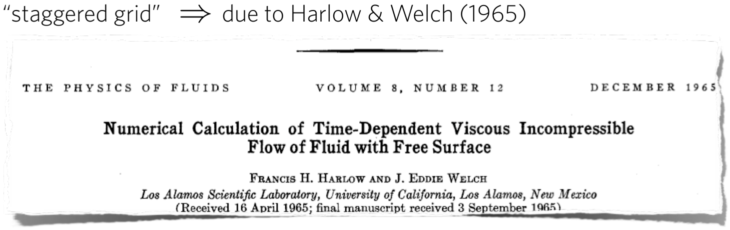
$$\frac{1}{2\Delta x}\left(\frac{-\Delta t}{\rho}\frac{p_{i+2}-p_i}{2\Delta x}+u_{i+1}^\star+\frac{\Delta t}{\rho}\frac{p_i-p_{i-2}}{2\Delta x}+u_{i-1}^\star\right)=0$$

- rewrite

$$\frac{p_{i+2}-2p_i+p_{i-2}}{4\Delta x^2}=\frac{\rho}{\Delta t}\frac{u_{i+1}^\star-u_{i-1}^\star}{2\Delta x}=0$$

Navier-Stokes **Solution**

- "staggered grid" $\Longrightarrow$ due to Harlow & Welch (1965)


THE PHYSICS OF FLUIDS VOLUME 8, NUMBER 12 DECEMBER 1965

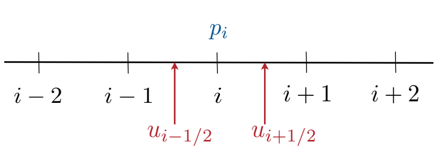
Numerical Calculation of Time-Dependent Viscous Incompressible Flow of Fluid with Free Surface

Francis H. Harlow and J. Eddie Welch

Los Alamos Scientific Laboratory, University of California, Los Alamos, New Mexico

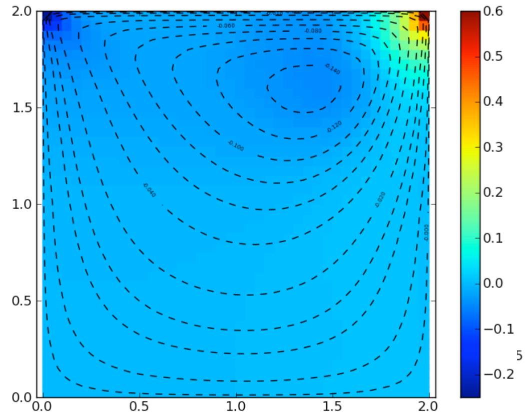
(Received 16 April 1965; final manuscript received 3 September 1965)

3825 citations on Google scholar, checked Mar.'13



Navier-Stokes **Collocated vs. Staggered grid**

- Cavity flow:



Step 11
41x41 mesh
dx=dy=0.05
nu=0.01 giving
Re=200