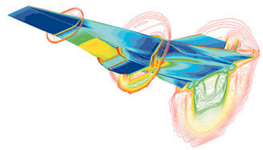
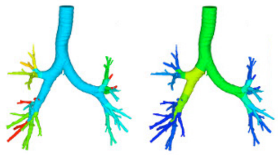


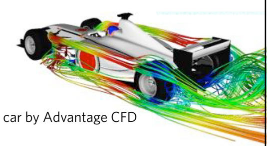
Hyper-X at Mach 7, NASA image in the public domain

ME 702
Computational Fluid Dynamics, CFD

Prof. Lorena A. Barba



Human airways, by FuiDA nv

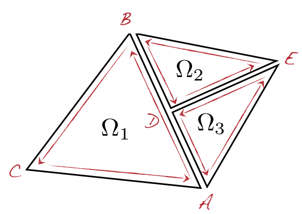


F1 car by Advantage CFD

BU

The FVM **Conservative discretization**

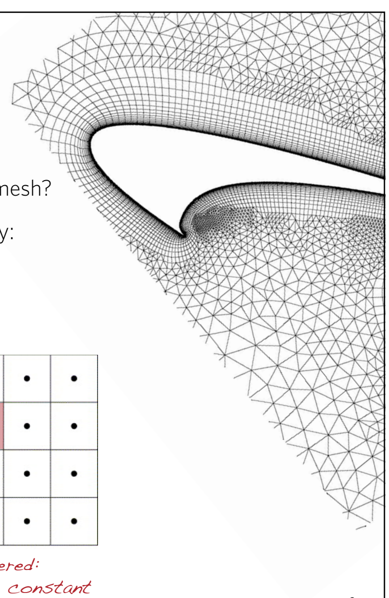
- The numerical flux is conserved from one discretization cell to the next.
  - Internal fluxes (numerical sources) appear in a non-conservative discretization.
- Good for solutions with strong gradients.
  - Continuous flows: numerical sources are of the order of the truncation error.
- More accurate.

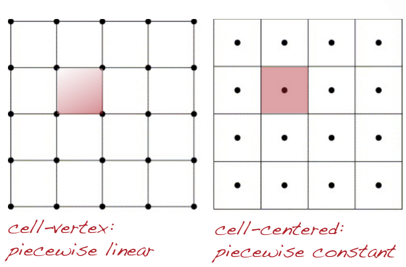


$$\frac{\partial}{\partial t}(\bar{U}_j\,\Omega_j) + \sum_{\text{faces}} \vec{F}\cdot\Delta\vec{S} = \bar{Q}_j\Omega_j$$

The FVM **Difficulties**

- Accurate definition of derivatives:
  - how to write Taylor expansions on this mesh?
- difficult to obtain higher-order accuracy:
  - two levels of approximation: integration & interpolation





The FVM **Approximation of derivatives**

- e.g., Navier-Stokes: viscous flux terms are functions of velocity gradients
- Gauss divergence theorem: $$\int_\Omega \vec{\nabla} U \,\mathrm{d}\Omega = \oint_S U \mathrm{d}\vec{S}$$
- define averaged gradients:

$$\left(\overline{\frac{\partial U}{\partial x}}\right)_\Omega \equiv \frac{1}{\Omega}\int_\Omega \frac{\partial U}{\partial x}\mathrm{d}\Omega = \frac{1}{\Omega}\int_S U\,\hat{i}\cdot \mathrm{d}\vec{S}$$

$$\left(\overline{\frac{\partial U}{\partial y}}\right)_\Omega \equiv \frac{1}{\Omega}\int_\Omega \frac{\partial U}{\partial y}\mathrm{d}\Omega = \frac{1}{\Omega}\int_S U\,\hat{j}\cdot \mathrm{d}\vec{S}$$

The FVM **MacCormack methods**

- Recall Lax-Wendroff for 1D convection, with: $a \equiv \dfrac{\partial f}{\partial u}$

$$u^{n+1} = u^n + \Delta t\left(-\frac{\partial f}{\partial x}\right)^n + \frac{\Delta t^2}{2}\frac{\partial}{\partial x}\left(a\frac{\partial f}{\partial x}\right)^n$$

- 2D version:

$$\vec{U}^{n+1} = \vec{U}^n + \Delta t\left(-\frac{\partial \vec{f}}{\partial x} - \frac{\partial \vec{g}}{\partial y}\right)^n + \frac{\Delta t^2}{2}\frac{\partial}{\partial x}\left(A^n\left(\frac{\partial \vec{f}}{\partial x} + \frac{\partial \vec{g}}{\partial y}\right)^n\right) + \frac{\Delta t^2}{2}\frac{\partial}{\partial y}\left(B^n\left(\frac{\partial \vec{f}}{\partial x} + \frac{\partial \vec{g}}{\partial y}\right)^n\right)$$

with the Jacobian matrices: $A \equiv \dfrac{\partial \vec{f}}{\partial \vec{U}}$ $\quad B \equiv \dfrac{\partial \vec{g}}{\partial \vec{U}}$

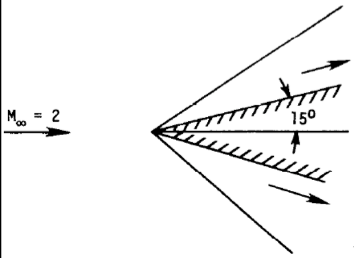
The FVM **For the Euler equations in 2D**

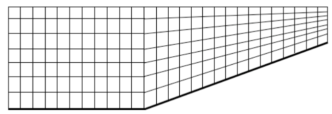
- FV formulation is possible since equations take form of a flux balance.

$$\frac{\partial \vec{U}}{\partial t} + \frac{\partial \vec{f}}{\partial x} + \frac{\partial \vec{g}}{\partial y} = 0$$

$$\vec{U} = \begin{bmatrix} \rho \\ \rho u \\ \rho v \\ \rho e \end{bmatrix} \qquad \vec{f} = \begin{bmatrix} \rho u \\ \rho u^2 + p \\ \rho u v \\ \rho h_T u \end{bmatrix} \qquad \vec{g} = \begin{bmatrix} \rho v \\ \rho u v \\ \rho v^2 + p \\ \rho h_T v \end{bmatrix}$$

FVM **Class project: wedge flow**



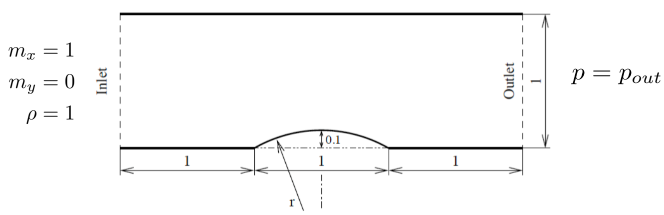


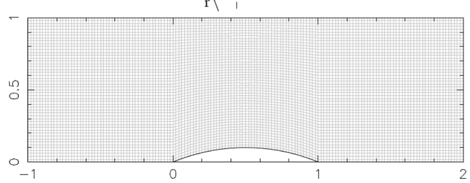
Free stream Mach	2.5
Wedge angle	15°
Shock angle	36.9°
Mach after shock	1.87
Pressure ratio	2.47
Density ratio	1.87
Temperature ratio	1.32

Analytic solution for the supersonic wedge flow

FVM **Class project: GAMM channel**

- test problem for transonic flows: use MacCormack, with artificial viscosity





$M_{inlet} = 0.675,\ \Delta t = 0.5$