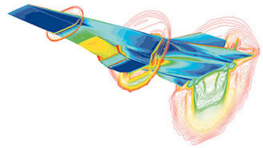
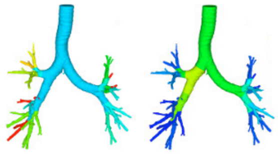
Hyper-X at Mach 7, NASA
image in the public domain

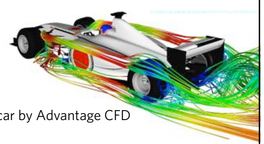


ME 702
Computational Fluid Dynamics, CFD

Prof. Lorena A. Barba



Human airways, by
FuiDA nv

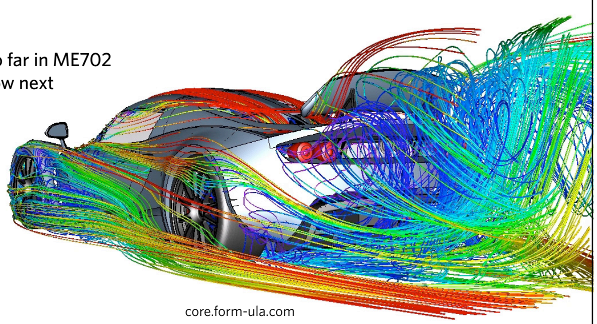


F1 car by Advantage CFD

BU

How to continue studying CFD?

Overview of
- what we've learned so far in ME702
- possible paths to follow next



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We've learned **Basic Finite Difference method**

- forward / backward / central
- truncation error / order of convergence
- stability / Lax equivalence theorem
- Modified differential equation / von Neumann stability analysis
- numerical schemes must be compatible with the physics
- physical interpretation of the CFL condition

Practicum **"12 steps to Navier-Stokes"**

- experience numerical diffusion
- experience "blowing things up"
- difference between linear / nonlinear convection: shocks
- Laplace / Poisson equations "relax" to BCs and sources on pseudotime
- pressure-Poisson equation and fractional step / issues with BCs
- visualizing solution as much work as getting solution!

Practicum **Schemes for convection: Burgers**

- Lax-Friedrichs / Lax-Wendroff / leapfrog / MacCormack
- explicit vs. implicit schemes
- using stencils / odd-even decoupling (connect back to Navier-Stokes on a collocated grid / staggered grid and famous MAC-method paper)
- dispersion errors & artificial diffusion
- spectral analysis of numerical schemes

Practicum **Euler equations**

- vector form / flux functions / concept of flux / conservation
- discretizing Euler equation with:
  - LF / LW / Richtmyer / McCormack
- Sod's test problems

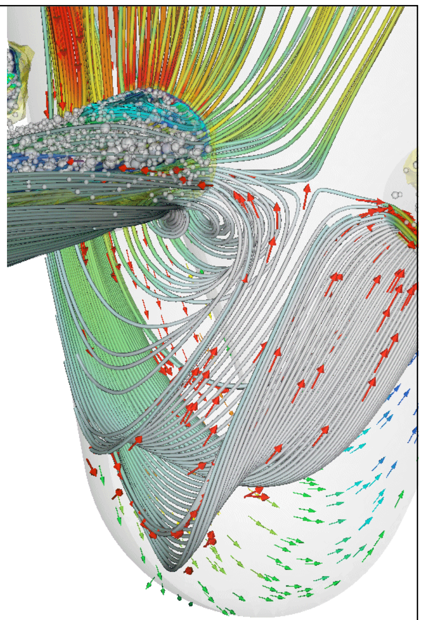
We've learned **Basic Finite Volume method**

- conservative discretization / integral formulation
- flexibility of general FV grids
- using classes to code FVM in a modular way
- difficulties: obtaining derivatives, higher orders of accuracy

Summary

1) Basic Finite-Difference method
2) Practicum: "12 steps to Navier-Stokes"
3) Practicum: schemes for convection
4) Practicum: Euler equations
5) Basic Finite-Volume method

What next?



Time integration methods

- Recall: $u_t = a u_x$
  different combinations of space / time schemes behave VERY differently
  - forward diff in time ⇒ unstable
  - backward diff in time + implicit ⇒ unconditionally stable
  - central diff in time (leapfrog) ⇒ conditionally stable
- **Given a space discretization**, what criteria to impose on time integration in order to obtain a stable & accurate scheme?

Time integration **General methodology**

1. Perform space discretization, express in matrix form (incl. BCs): $\dfrac{\mathrm{d}\vec{U}}{\mathrm{d}t} = S \cdot \vec{U} + Q$
2. Perform spectral analysis of the matrix S (eigenvalue spectrum)
3. Stability conditions:
   - exact solution of system of ODEs should not grow unbounded
   - eigenvalues of S must have non-positive real parts
4. Select time integration method and define its stability, as fnc. of eigenvalues of S

- Example (1) — Diffusion $\quad u_t = \alpha u_{xx}$

using CD in space:

$$\frac{\mathrm{d}u_i}{\mathrm{d}t} = \frac{\alpha}{\Delta x^2}\left(u_{i+1} - 2u_i + u_{i-1}\right)$$

the semi-discretized form

with Dirichlet BCs:

$$\frac{\mathrm{d}U}{\mathrm{d}t} = \frac{\alpha}{\Delta x^2}\begin{vmatrix} -2 & 1 & \cdot & \cdot & & & \cdot \\ 1 & -2 & 1 & & & & \cdot \\ & 1 & -2 & 1 & & & \\ & & \cdot & \cdot & \cdot & & \\ & & & & \cdot & & \\ & & & & 1 & -2 & 1 \\ \cdot & & & & & 1 & -2 \end{vmatrix}\begin{vmatrix} u_1 \\ \cdot \\ u_{i-1} \\ u_i \\ u_{i+1} \\ \cdot \\ u_{N-1} \end{vmatrix} + \begin{vmatrix} a\alpha/\Delta x^2 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ b\alpha/\Delta x^2 \end{vmatrix}$$

with Neumann BCs

$$\frac{\mathrm{d}U}{\mathrm{d}t} = \frac{\alpha}{\Delta x^2}\begin{vmatrix} -1 & 1 & \cdot & \cdot & & & \cdot \\ 1 & -2 & 1 & & & & \cdot \\ & 1 & -2 & 1 & & & \\ & & \cdot & \cdot & \cdot & & \\ & & & & \cdot & & \\ & & & & 1 & -2 & 1 \\ \cdot & & & & & 1 & -2 \end{vmatrix}\begin{vmatrix} u_1 \\ \cdot \\ u_{i-1} \\ u_i \\ u_{i+1} \\ \cdot \\ u_{N-1} \end{vmatrix} + \begin{vmatrix} -a\alpha/\Delta x \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ b\alpha/\Delta x^2 \end{vmatrix}$$

the form of the matrix depends on the type of BCs

- study eigenvalue spectrum
- various time-stepping schemes (ODE integrators): Euler method, Adams-Bashford, Runge-Kutta ... and their stability regions

Iterative solvers

- We obtain systems of algebraic equations from:
  - applying implicit time integration schemes to time-dependent problems
  - space discretizations of steady-state formulations
- direct methods of solution are too expensive, $\mathcal{O}(N^3)$
- iterative methods:
  - point-Jacobi / Gauss-Seidel / successive-over-relaxation, SOR / line relaxation
  - convergence of iterative solvers / preconditioning techniques
  - the multigrid method

Advanced CFD topics

- shock-capturing methods / flux-splitting schemes (e.g., van Leer)
  flux-difference splitting (Roe scheme)
- higher-order schemes
- turbulence models