HOMEWORK 3: MATH 183 WINTER 2013 (UPDATED VERSION)

DUE IN CLASS ON FRIDAY JAN 25TH

Problem 1 is Exercise 2.12 in OpenIntroStatistics book (p. 109).

Problem 2. Two people each toss a coin n times. Find probability that they will toss the same number of heads ?

Problem 3. When coded messages are sent, there are sometimes errors in transmission. In particular, morse code uses "dots" and "dashes", which are known to occur in proportion 3 : 4. This means that for any given symbol

$$P(\text{dot sent}) = \frac{3}{7} \text{ and } P(\text{dash sent}) = \frac{4}{7}.$$

Suppose there is inference on the transmission line, and with probability 1/8 a dot it mistakenly received as a dash and vice versa. If we receive a dot, can we be sure that a dot was sent ?

Problem 4. A ball is drawn at random from an urn containing one red and one white ball. If the white ball is drawn, it is put back into the urn. If the red ball is drawn, it is returned to the urn together with two more red balls. Then a second draw is made. What is the probability a red ball was drawn on both the first and second draws?

Problem 5. In early 2001, the European commission introduced massive testing of cattle to determine infection with the transmissible form of Mad Cows Disease (MCD). As not test is 100% accurate, most tests have the problem of false positives and false negatives. A false positive means that according to the test the cow is infected, but in actuality it is not. A false negative means an infected cow is not detected by the test.

Imagine we test cow. Let B denote the event "the cow has MCD" and T the event "the test comes up positive" (test jargon for: according to the test we should believe the cow is infected with MCD). One can 'test the test" by analyzing samples from cows that are known to be infected or known to be health and so determine the effectiveness of the test.

The European Commisiion had this done for four tests in 1999 and for several more later. The results for what the creport calls Test A may be summarized as follows: an infected cow has 70% chance of testing positive, and healthy cow just 10%.

(a) Determine the probability that an arbitrary cow tests positive if it was estimated that there is 2% chance that the cow has MCD?

(b) Another more pertinent question is: suppose my cow tests positive. What is the probability it really has MCD ?

Problem 6. Calculate the following:

(a) The events A, B, C satisfy $P(A|B \cap C) = 1/4$, $P(B|C) = 1/3$ and $P(C) = 1/2$. Calculate $P(A^c \cap B \cap C)$.

(b) Two independent events A and B are given and $P(B|A \cup B) = 2/3$, $P(A|B) = 1/2$. What is $P(B)$?

(c) If $P(A) = \frac{1}{3}$ and $P(B^c) = \frac{1}{4}$, can A and B be disjoint ? Explain!

Problem 7 is Exercise 2.21 in OpenIntroStatistics book (p. 112).

Problem 8 is Exercise 2.24 in OpenIntroStatistics book (p. 112).

Problem 9. Let X represent the number of times 6 appeared in two independent throws of a die.

(a) Describe probability distribution of X, by giving either probability mass function of cumulative distribution function.

(b) Definite events

$A = \{$maximum in the two throws was 2 and X=0$\}$,

$B = \{$sum of the two throws was 5 and X=1$\}$,

$C = \{$maximum in the two throws was 8 and X=1$\}$.

What are their probabilities ?

(c) Are the events $D = \{$X=0$\}$ and $E = \{$maximum in the two throws was 2$\}$ independent ?

Problem 10. Let X be discrete random variable with probability mass function given by

x	-1	0	1	2
P(X=x)	1/4	1/8	1/8	a

and $P(X = x) = 0$ for all other x's.

(a) Find a so that previous table defined probability distribution.
(b) Let random variable $Y = X^2$. Calculate probability mass function of Y.
(c) Calculate cumulative probability distribution function of X and Y for $x = 3/4$ and $x = \pi - 3$.

Problem 11. Probability density function f of a continuous random variable X is given by

$$f(x) = \begin{cases} cx + 3, & , -3 \leq x \leq -2 \\ 3 - cx, & 2 \leq x \leq 3 \\ 0, & \text{otherwise} \end{cases}$$

(a) Find c.
(b) Calculate cumulative probability distribution function of X.

Challenge Problem: Produce probability numbers of the "game show" example from lecture by proper calculation and simulation in R.