`Problem 1`

(a) In this whole problem, let's denote miss k days by k. Then, $P(k=0) = 1 - P(k=1) - P(k=2) - P(k \geq 3) = 1 - 0.25 - 0.15 - 0.28 = .32$

(b) $P(k \leq 1) = P(k=0) + P(k=1) = 1 - P(k=2) - P(k \geq 3) = 1 - 0.15 - 0.28 = 0.57$

(c) $P(k \geq 1) = 1 - P(k=0) = 1 - 0.32 = 0.68$

(d) P(neither miss any school) $= P$(first misses no school) $\times P$(second misses no school) $= P(k=0) \times P(k=0) = 0.32^2$. Note that we assumed that the two events "first misses no school" and "second misses no school" were independent!

(e) P(both miss at least one) $= P$(first miss at least one) $\times P$(second miss at least one) $= P(k \geq 1) \times P(k \geq 1) = 0.68^2$. Again, we assumed that the two events were independent.

(f) We assumed whether a child missing school or not was independent of his/her sibling missing school or not. This is probably not true, since children living together can get sick because of each other. However, our answer could still be somewhat close to the correct answer.

`Problem 2`

For this problem, we have the following 2 methods:

I. Let's say two people are A and B. then event {A and B toss the same number of heads} = $\bigcup_{k=0}^{n}$ {A and B both get k heads}

As the events {A and B get k heads} are disjoint mutually, we have:

$$
\begin{aligned}
P\bigcup_{k=0}^{n}\{\text{A and B both get k heads}\} &= \sum_{k=0}^{n} P\{\text{A and B both get k heads}\} \\
&= \sum_{k=0}^{n}\left[\binom{n}{k}\left(\frac{1}{2}\right)^{k}\left(\frac{1}{2}\right)^{n-k}\right]^{2} \\
&= \left(\frac{1}{4}\right)^{n}\sum_{k=0}^{n}\binom{n}{k}^{2} \text{ (you can leave the result like that)}
\end{aligned}
$$

If you are interested in how to simplify the answer, see the following.

$$\begin{aligned}\left(\frac{1}{4}\right)^n \sum_{k=0}^{n} \binom{n}{k}^2 &= \left(\frac{1}{4}\right)^n \sum_{k=0}^{n} \binom{n}{k}\binom{n}{n-k} \\ &= \left(\frac{1}{4}\right)^n \binom{2n}{n}\end{aligned}$$

where we are using Vandermonde's indentity:

$$\binom{m+n}{r} = \sum_{k=0}^{r} \binom{m}{k}\binom{n}{r-k}$$

II. As the coin is fair, we will find that, $P\{\text{A gets k Heads}\} = P\{\text{A gets k Tails}\} = P\{\text{A gets n-k Heads}\}$. That means A and B totally toss coin for 2n times, and the question we want is equal to the probability of A and B totally get n Heads, which is

$$\binom{2n}{n}\left(\frac{1}{4}\right)^n$$

`Problem 3`

Let $A = \{\text{a dot is sent}\}$, then $A^C = \{\text{a dash is sent}\}$. Similarly, let $B = \{\text{a dot is received}\}$, then $B^c = \{\text{a dash is received}\}$.

Then to make a decision when we receive a dot, we are interested $P(A|B)$.

From the assumption in problem, we have $P(A) = \frac{3}{7}, P(A^c) = \frac{4}{7}, P(B^c|A) = \frac{1}{8}, P(B|A^c) = \frac{1}{8}$, and so $P(B|A) = \frac{7}{8}$.

By Bayes formula, we have:

$$\begin{aligned}P(A|B) = \frac{P(A\bigcap B)}{P(B)} &= \frac{P(B|A)P(A)}{P(B\bigcap A) + P(B\bigcap A^c)} \\ &= \frac{P(B|A)P(A)}{P(B|A)P(A) + P(B|A^c)P(A^c)} \\ &= \frac{7/8 \cdot 3/7}{7/8 \cdot 3/7 + 1/8 \cdot 4/7} \\ &= \frac{21/56}{25/56} \\ &= \frac{21}{25}\end{aligned}$$

`Problem 4`

Let A be {draw a red ball at the first time}, let B be {draw a red ball at the second time}. As the assumption in problem, we have:

$$P(A \bigcap B) = P(B|A)P(A) = \frac{3}{4} \cdot \frac{1}{2} = \frac{3}{8}$$

Problem 5

(a) Let A be {an arbitrary cow has MCD}, let B be {the test of an arbitrary cow is positive}. Then we know $P(A) = 2\%, P(B|A) = 70\%, P(B|A^c) = 10\%$. Then,

$$\begin{aligned} P(B) &= P(B \bigcap A) + P(B \bigcap A^c) \\ &= P(B|A)P(A) + P(B|A^c)P(A^c) \\ &= 70\% \cdot 2\% + 10\% \cdot 98\% \qquad = 11.2\% \end{aligned}$$

(b)

$$\begin{aligned} P(A|B) &= \frac{P(A \bigcap B)}{P(B)} \\ &= \frac{P(B|A)P(A)}{P(B)} \\ &= \frac{70\% \cdot 2\%}{11.2\%} \\ &= 12.5\% \end{aligned}$$

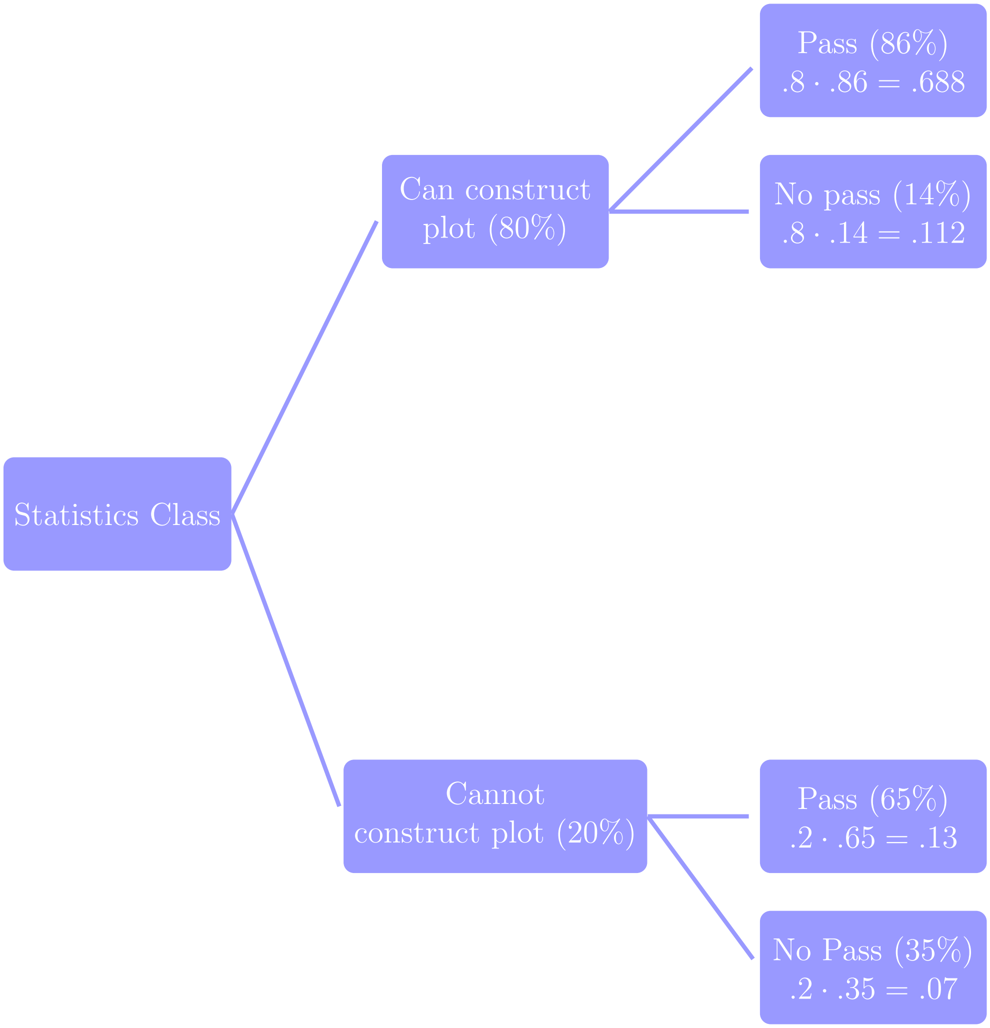
Problem 6

(a) $P(A^c \cap B \cap C) = P(C) \times P(B|C) \times P(A^c|B \cap C) = \frac{1}{2} \times \frac{1}{3} \times P(A^c|B \cap C) = \frac{1}{2} \times \frac{1}{3} \times (1 - P(A|B \cap C)) = \frac{1}{2} \times \frac{1}{3} \times \frac{3}{4}$. The last equality follows from the fact that a conditional probability function, ie something of the form $P(-|D)$, is itself a probability function, and hence satisfies all the properties of probability functions.

(b) Since A and B are independent, $P(A|B) = \frac{1}{2}$ tells us that $P(A) = \frac{1}{2}$. Now, $\frac{2}{3} = P(B|A \cup B) = \frac{P(B \cap (A \cup B))}{P(A \cup B)} = \frac{P(B)}{P(A \cup B)} = \frac{P(B)}{P(A) + P(B) - P(A \cap B)} = \frac{P(B)}{0.5 + P(B) - 0.5P(B)} = \frac{P(B)}{0.5 + 0.5P(B)}$. Hence, we rearrange to get $1 + P(B) = 3P(B)$, so $P(B) = \frac{1}{2}$.

(c) $P(B^c) = \frac{1}{4}$ means $P(B) = \frac{3}{4}$. If A and B were disjoint, we'd have $P(A \cup B) = P(A) + P(B) = \frac{1}{3} + \frac{3}{4} > 1$. That can't be true.

Problem 7



(a)

(b)

$$P(\text{construct}|\text{pass}) = \frac{P(\text{construct}) \cap \text{pass})}{P(\text{pass})} = \frac{.688}{.688 + .13} \approx .841$$

Problem 8

Let's write out the facts they give us in terms of probabilities. P(vote for Walker) = 0.53. P(college degree | vote for Walker) = 0.37 and P(college degree | not vote for Walker) = 0.44. The question asks P(vote for Walker | college degree). By Bayes' theorem, that is

$$= \frac{P(\text{college degree}|\text{vote W})P(\text{vote W})}{P(\text{college degree}|\text{vote W})P(\text{vote W}) + P(\text{college degree}|\text{not vote W})P(\text{not vote W})} =$$

$$= \frac{0.37 \times 0.53}{0.37 \times 0.53 + 0.44 \times 0.47}$$

Problem 9

(a) For X to equal 2, we must have a six rolled both times, so there is a $\frac{1}{6} \cdot \frac{1}{6}$ chance of this occuring. For X to equal 0 we must have a six **not** rolled both times, so there is a $\frac{5}{6} \cdot \frac{5}{6}$ chance of this occuring. $X = 1$ must be the remaining cases, so we use the law of total probability

$$P(X=0) = \frac{25}{36}$$
$$P(X=1) = 1 - \frac{1}{36} - \frac{25}{36} = \frac{10}{36}$$
$$P(X=2) = \frac{1}{36}$$

(b) For A to occur, we could have rolled 1-2, 2-1, or 2-2, so we have

$$P(A) = \frac{3}{36}.$$

For B, if $X = 1$, this means a 6 was rolled, and the sum must be greater than or equal to 7. Thus this event cannot occur and

$$P(B) = 0.$$

For C, if $X = 1$ then a 6 was rolled, and for the sum to be 8 the other roll must be a 2. Thus we may have 2-6 or 6-2 for C to occur, and so

$$P(C) = \frac{2}{36}.$$

If C is interpreted as the maximum roll of a single dice is 8, then it has probability 0 as dice only go up to 6.

(c) Events D and E are not independent. By part (b) the probability of $D \cap E$ is $\frac{3}{36}$. However,

$$P(D) \cdot P(E) = \frac{25}{36} \cdot \frac{3}{36} \neq P(D \cap E).$$

Problem 10

(a) All we need a to satisfy in order to have a probability distribution is that it be positive and result in a total probability of 1. So we have $\frac{1}{4} + \frac{1}{8} + \frac{1}{8} + a = 1$, giving us $a = \frac{1}{2}$.

(b) What values can $Y = X^2$ take? Only the squares of values that X took, so only 0, 1, and 4. Now we compute:

$P(Y = 0) = P(X^2 = 0) = P(X = 0) = \frac{1}{8}$.

$P(Y = 1) = P(X^2 = 1) = P(X = 1 \cup X = -1) = P(X = 1) + P(X = -1) = \frac{1}{8} + \frac{1}{4} = \frac{3}{8}$.

$P(Y = 4) = P(X^2 = 4) = P(X = 2) = a = \frac{1}{2}$.

(c) $F_X(\frac{3}{4}) = P(X \leq \frac{3}{4}) = P(X = -1) + P(X = 0) = \frac{1}{4} + \frac{1}{8}$

$F_Y(\frac{3}{4}) = P(Y \leq \frac{3}{4}) = P(Y = 0) = \frac{1}{8}$

$F_X(\pi - 3) = P(X \leq \pi - 3) = P(X = -1) + P(X = 0) = \frac{1}{4} + \frac{1}{8}$

$F_Y(\pi - 3) = P(Y \leq \pi - 3) = P(Y = 0) = \frac{1}{8}$