MATH 183

SOLUTIONS TO THE PRACTICE MIDTERM I
MATHEMATICS DEPARTMENT

JELENA BRADIC

1.

(a) Correct answers are

(3) as from contingency tables one can read frequency of one category in the same which is center of discrete distribution

(5) as this is precisely the definition of probability from slides of Chapter 2

(b) Correct answers are

(1) as this is the definition of histogram whereas boxplots are missing modes for example

(c) Correct answer is

(2) as two dice are independent of each other hence one can view this as repeated experiment of one dice

2. These are approximations

(a) \$35000
(b) \$40000
(c) \$20000
(d) \$50000 - \$70000
(e) \$20000 - \$30000
(e) \$30000 - \$40000
(g) 2:5%*10 + 2%*10 + 1%*10= 55%

3.

(a)

$$\frac{\binom{10}{2}\binom{6}{2}\binom{4}{1}}{\binom{20}{5}}$$

(b)

$\mathbb{P}$ (takes at least 4 tosses) =

$\mathbb{P}$ (There is at most one HEAD in the first 3 tosses) =

$\mathbb{P}$ (No HEAD in the first 3 tosses) + $\mathbb{P}$(exactly one HEAD in the first 3 tosses) =

$\frac{1}{2}^3 + 3 * \frac{1}{2}^3 = 0.5.$

4.

(a) Let B be the event that the ball was taken. Since this event will depend on whether roll of a die resulted in $\{5, 6\}$ or $\{1, 2, 3, 4\}$ we will condition out on both outcomes. Hence, let D_1 be the event that roll of a die resulted in $\{5, 6\}$ and similarly let D_2 be its opposite event, i.e. roll of a die resulted in $\{1, 2, 3, 4\}$. Then we know

$$P(D_1) = 2/6, \quad P(D_2) = 4/6$$

So,

$$P(B) = P(B|D_1) * P(D_1) + P(B|D_2) * P(D_2) = \frac{2}{5} * \frac{2}{6} + \frac{5}{7} * \frac{4}{6} = 0.609$$

(b)

$$P(D_1|B) = \frac{P(B|D_1) * P(D_1)}{P(B)} = \frac{\frac{2}{5} * \frac{2}{6}}{\frac{2}{5} * \frac{2}{6} + \frac{5}{7} * \frac{4}{6}}$$

5.

(a) c is a solution of the following equation

$$\int_{-3}^{-1} (cx^2 + 3)dx + \int_{1}^{3} (2 - cx)dx = 1$$

Left hand side is then equal to

$$c\frac{26}{3} + 3 * 2 + 2 * 2 - c4 = c\frac{14}{3} + 10$$

hence $c = -27/14$.

(b) Note that previous density function is not quite a density function as it is not positive in all points. Take $f_X(-3)$ which is much smaller then zero.

But, if it was a proper density, here is what we should have done Let us denote CDF of X with $F_X(x)$. Then by following definition of $F_X(x) = \int_{-\infty}^{x} f_X(u)du$ for density function f_X we have

$$F_X(x) = \begin{cases} 0 & \text{if} \quad x < -3 \\ \int_{-3}^{x} (-\frac{27}{14}u^2 + 3)du & \text{if} \quad -3 \le x \le -1 \\ \int_{-3}^{-1} (-\frac{27}{14}u^2 + 3)du & \text{if} \quad -1 \le x \le 1 \\ \int_{-3}^{-1} (-\frac{27}{14}u^2 + 3)du + \int_{1}^{x} (2 + \frac{27}{14}u)du & \text{if} \quad 1 \le x \le 3 \\ 1 & \text{if} \quad x > 3 \end{cases}$$

(c) If f_X was proper density then we would do the following

$$P(-2 \le X \le 3) = F_X(3) - F_X(-2) = 1 - F_X(-2) = 1 - \int_{-3}^{-2} (-\frac{27}{14}u^2 + 3)du$$