Ex: let $X_1, X_2, \dots, X_n$ be sample FROM POISSON DISTR. WITH PARAMETER λ. (UNKNOWN)

a) FIND MLE ESTIMATOR OF λ.

b) FIND MLE ESTIMATE OF λ IF $X_1 = 3, X_2 = 5, X_3 = 4, X_4 = 2$.

a) Step 1: FIND pmf OF X_i's

$$p_X(x;\lambda) = \frac{e^{-\lambda}(\lambda)^x}{x!} = P(X = x)$$

Step 2: FIND $L(\lambda)$ (likelihood function)

$$L(\lambda) = \prod_{i=1}^{n} P(X = x_i) = \prod_{i=1}^{n} \frac{e^{-\lambda} \cdot (\lambda)^{x_i}}{(x_i)!}$$

$$= \frac{e^{-\lambda}\cdot(\lambda)^{x_1}}{(x_1)!} \cdot \frac{e^{-\lambda}\cdot\lambda^{x_2}}{(x_2)!} \cdot \frac{e^{-\lambda}\,\lambda^{x_3}}{(x_3)!} \cdots \frac{e^{-\lambda}\,\lambda^{x_n}}{(x_n)!}$$

$$\cancel{(n \cdot e^{-\lambda})}$$

$$= e^{-n\lambda} \cdot \lambda^{(x_1 + x_2 + \dots + x_n)} \cdot \prod_{i=1}^{n} \frac{1}{(x_i)!}$$

Step 3: Maximize $L(\lambda)$.

a) Take log of L

$$\text{Log } L(\lambda) = \log\left(e^{-n\lambda} \cdot \lambda^{\sum_{i=1}^{n} x_i} \cdot \prod_{i=1}^{n} \frac{1}{(x_i)!}\right).$$

Remember: $\log(a \cdot b) = \log a + \log b$

$$\log L(\lambda) = \log\left(e^{-n\lambda} \cdot \lambda^{\left(\sum_{i=1}^{n} x_i\right)} \cdot \prod_{i=1}^{n} \frac{1}{(x_i)!}\right).$$

$$= \log_e\left(e^{-n\lambda}\right) + \log\left(\lambda^{\left(\sum x_i\right)}\right) + \underbrace{\log\left(\prod_{i=1}^{n} \frac{1}{(x_i)!}\right)}.$$

$$= -n\cdot\lambda + \left(\sum_{i=1}^{n} x_i\right)\cdot \log \lambda + \underline{\text{constant}}$$

Step 3: (b).

$$\cancel{\frac{\partial}{\partial \lambda} \log L(\lambda) = 0}.$$

$$\frac{d}{d\lambda}\left(\log L(\lambda)\right) = 0.$$

$$-n + \frac{\left(\sum_{i=1}^{n} x_i\right)}{\lambda} = 0 \Rightarrow \hat{\lambda} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

$$\hat{\lambda} = \bar{X}$$

Ex (b) $X_1 = 3$, $X_2 = 5$, $X_3 = 4$, $X_4 = 2$

Estimate is $\hat{\lambda}\big|_{data} = \frac{3+5+4+2}{4} = 3.5$

EX: Let $X_1, X_2, \dots, X_n$ be a sample from continuous distribution with pdf as follows

$$f_X(x,\theta) = e^{-(x-\theta)}, \text{ for } \underbrace{x \geq \theta, \text{ and } \theta > 0}$$

(a) Find MLE estimator of θ?

(c) Is it unbiased estimator?

(b) What is the distribution of MLE estimator?

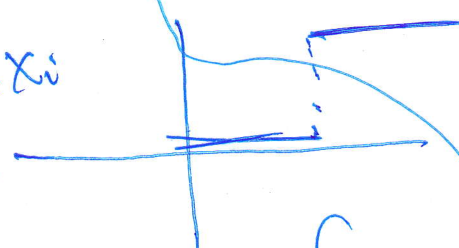
Step 1: $f_X(x;\theta) = e^{-(x-\theta)}$, for $x \geq \theta$ and $\theta \geq 0$.

$0 < \theta < X$

Step 2: $L(\theta) = \prod_{i=1}^{n} f_X(x_i;\theta)$

$= \prod_{i=1}^{n} \left(e^{-(x_i-\theta)} \cdot \mathbb{1}(0<\theta\le x_i) \right)$.

$$\mathbb{1}(0<\theta<x_i) \overset{def}{=} \begin{cases} 1 & \text{if } 0<\theta\le x_i \\ 0 & \text{if o.w.} \end{cases}$$



$$\mathbb{1} = f(\theta) = \begin{cases} 1, \\ 0, \end{cases}$$

$$L(\theta) = e^{-\sum_{i=1}^{n} x_i} \cdot e^{n\theta} \cdot \underbrace{\prod_{i=1}^{n} \mathbb{1}(0<\theta\le x_i)}_{\mathbb{1}(0<\theta\le x_i \text{ for all } i)}$$

$$= \mathbb{1}\left(0<\theta\le \min_{1\le i\le n} x_i\right)$$