HOMEWORK 8: MATH 183 WINTER 2013

DUE IN CLASS ON FRIDAY MARCH 8TH

`Reading: Chapters 5.3 & 5.5 of Larsen & Marx.`

`Reading: 7.4 (first two topics only) & 7.5 (first two topics only) of Larsen & Marx.`

(1) [Problem 1 of Midterm II]

Automobiles arrive at a vehicle equipment inspection station according to a Poisson process with a rate of $\lambda = 10$ per hour. Suppose that with probability 0.5 an arriving vehicle will have no equipment violations.

(a) What is the probability that exactly 5 have no violations?
(b) What is the probability that exactly ten arrive during the next hour and all ten have no violations?
(c) For any fixed $y \geq 10$, show that the probability that y vehicles arrive during the next hour, of which ten have no violations, is given by

$$\frac{e^{-10}5^y}{10!(y-10!)}$$

(2) [Problem 2 of Midterm II]

Let $X_1, ..., X_n$ be i.i.d random variables with uniform distribution on $[2\theta, 10]$ In particular, each of them has density function

$$f_X(x;\theta) = \frac{1}{10-2\theta} \quad \text{for} \quad 2\theta \leq x \leq 10$$

(a) Write down the Likelihood function $L(\theta)$ and give its simplest form
(b) Find maximum likelihood estimator of θ. Call it $\hat{\theta}$.
(c) Find the CDF of $\hat{\theta}$
(d) Find the PDF of $\hat{\theta}$
(e) Write up the equation for the expected value of $\hat{\theta}$?

(3) [Problem 3 of Midterm II]

The time that customers take to complete their transaction at a money machine is a random variable with mean = 2 minutes and standard deviation = 0.6 minutes. About 30% of customers take more than 3 minutes to complete their transaction. Take a random sample of size 50. Find the probability that

(a) the selected sample takes on average between 1.8 minutes and 2.25 minutes
(b) more than 34% of the selected people take more than 3 minute to complete all their transactions.

[Hint: remember central limit theorem]

(4) Explain your answers fully

(a) Derive approximate double sided 98% Confidence interval for p in the Binomial distribution $\mathcal{B}(n,p)$ when n is known [Hint:done in lecture]

(b) Induce a definition of one-sided Confidence interval of level $100\%(1-\alpha)$, given the definition of two sided confidence interval for p in the Binomial distribution $\mathcal{B}(n,p)$ when n is known.

(c) Explain which of the following candidates are and which are not the correct one, one-sided Confidence interval of level $100\%(1-\alpha)$ for p in the Binomial distribution $\mathcal{B}(n,p)$ when n is known:

(1)
$$P\left(\frac{\hat{p}-p}{\sqrt{p(1-p)/n}} > z_\alpha\right) = 1-\alpha$$

(2)
$$P\left(\frac{\hat{p}-p}{\sqrt{p(1-p)/n}} \le z_{1-\alpha}\right) = 1-\alpha$$

(3)
$$P\left(\frac{\hat{p}-p}{\sqrt{p(1-p)/n}} > z_{\alpha/2}\right) = 1-\alpha$$

(4)
$$P\left(-z_{\alpha/2} \le \frac{\hat{p}-p}{\sqrt{p(1-p)/n}} \le z_{\alpha/2}\right) = 1-\alpha$$

(d) Derive approximate one-sided 98% Confidence interval for the Binomial distribution $\mathcal{B}(n,p)$ when n is known [Hint: follow the procedure done in class and pay attention where does it differ compared to part (a)]

Solve the following exercises in full from Larsen and Marx textbook.

(5) 5.3.6.

(6) 5.3.22.

(7) 5.5.2

Challenge problem:

(a) Find MLE estimator of the var(X) for the problem 2.

(b) Is it an unbiased estimator?