

Wg: ~~Wednesday~~ Monday

jbradic

Microsoft Word - Math 183 - HW 6.docx

[21/Feb/2013:12:31:44 -0800] - p5880c-238476 - math64.ucsd.edu

CONFIDENCE INTERVALS (Interval estimation).

→ $X_1, \dots, X_n \sim f_\theta(x)$ for some parameter θ

MLE leads to $\hat{\theta}$ - NO INFORMATION ON
INTRINSIC RELIABILITY.

We want $[L(\text{Data}), U(\text{Data})]$ which contains
true θ with probability $(1-\alpha)$.

$$\mathbb{P}(\theta \in [L(\text{Data}), U(\text{Data})]) = 1-\alpha. \quad (*) \text{ def CI.}$$

EX: $X \sim \text{Bin}(n, p)$

$\hat{p} = \frac{X}{n}$ - MLE estimator of binomial
parameter p .

$$\frac{\hat{p} - \mathbb{E}\hat{p}}{\sqrt{\text{var}(\hat{p})}} \rightarrow \mathcal{N}(0, 1)$$

$$\mathbb{E}\hat{p} = \mathbb{E}\left(\frac{X}{n}\right) = p.$$

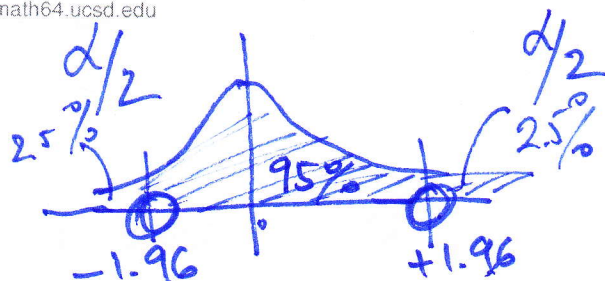
$$\text{var} \hat{p} = \text{var}\left(\frac{X}{n}\right) = \frac{1}{n^2} \text{var}(X) = \frac{1}{n^2} \cdot n \cdot p(1-p) = \frac{p(1-p)}{n}$$

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[21/Feb/2013:12:31:27 -0800] - p5880c-238475 - math64.ucsd.edu

$$\frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}} \rightarrow Z(0,1)$$



$$P\left(-Z_{\alpha/2} \leq \frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}} \leq Z_{\alpha/2}\right) = 1 - \alpha.$$

↓ isolate p: too difficult quadratic inequality

It is justifiable to replace p by $\hat{p}$ in the denominator.

(as $\hat{p} \rightarrow p$ in some strong sense)

Hence

$$P\left(-Z_{\alpha/2} \leq \frac{\hat{p} - p}{\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}} \leq Z_{\alpha/2}\right) = 1 - \alpha$$

↓ isolate p.

$$P\left(\hat{p} - Z_{\alpha/2} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \leq p \leq \hat{p} + Z_{\alpha/2} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right) = 1 - \alpha$$

$$\Rightarrow CI_p = \left[\hat{p} - Z_{\alpha/2} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}, \hat{p} + Z_{\alpha/2} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \right]$$

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HW6_Richard_Tominaga_A10582389.pdf

[21/Feb/2013:12:30:42 -0800] - p5880c-238474 - math64.ucsd.edu

MARGIN OF ERROR $\rightarrow$

$$\text{is } \frac{2}{2} \times \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \quad (\text{radius of CI})$$

How to choose sample size to get a $(1-\alpha)$ -CI to have radius ε (i.e. to be reasonably sure that $\hat{p}$ is within ε of p).

$$\varepsilon = z_{1-\frac{\alpha}{2}} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$\Rightarrow n = \frac{4\hat{p}(1-\hat{p})}{\varepsilon^2}$$

|| FOR p GETS SHORTER WHEN p IS NEAR 0 OR 1 (CLT is less reliable in such cases).

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[21/Feb/2013:12:30:30 -0800] - p5880c-238473 - math64.ucsd.edu

$$X_1, \dots, X_n \sim \mathcal{N}(\mu, \sigma^2)$$

$\uparrow$ $\uparrow$
 unknown unknown.

$$Cl_{\alpha}(\mu) = ?$$

$$\hat{\mu} = \bar{X}$$

$$\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \rightarrow \mathcal{Z}(0, 1)$$

BUT WE DON'T KNOW σ .

Replace σ with $\hat{\sigma}$ IN THE denominator.
 random variable

$$\frac{\bar{X} - \mu}{\hat{\sigma}/\sqrt{n}} \sim t_{n-1} \quad \text{student t distribution}$$

$\uparrow$
 random variable

$$\hat{\sigma}^2 = \frac{1}{n-1} \sum_{i=1}^n (X_i - \bar{X})^2 \quad \# \hat{\sigma}^2 = \hat{\sigma}^2$$

$$\# \left(\frac{1}{n} \sum_{i=1}^n (X_i - \bar{X})^2 \right) = \frac{n-1}{n} \sigma^2$$

$$\mathbb{P} \left(-t_{\alpha/2, n-1} \leq \frac{\bar{X} - \mu}{\sqrt{\hat{\sigma}^2/n}} \leq t_{\alpha/2, n-1} \right) = 1 - \alpha$$

$\downarrow$ isolate μ

$$\mathbb{P} \left(\bar{X} - t_{\alpha/2, n-1} \cdot \frac{\hat{\sigma}}{\sqrt{n}} \leq \mu \leq \bar{X} + t_{\alpha/2, n-1} \cdot \frac{\hat{\sigma}}{\sqrt{n}} \right) = 1 - \alpha$$

$$CI = \left[\bar{X} - \frac{\hat{\sigma}}{\sqrt{n}} \cdot t_{\alpha/2, n-1}, \bar{X} + t_{\alpha/2, n-1} \cdot \frac{\hat{\sigma}}{\sqrt{n}} \right)$$

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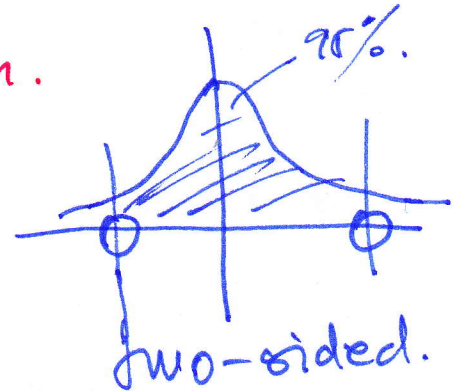
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[21/Feb/2013:12:30:02 -0800] - p5880c-238472 - math64.ucsd.edu

$X_1, \dots, X_n \sim N(\mu, \sigma^2)$
 unknown ~~parameter~~ unknown.

$C_{\sigma^2} = ?$

$$\boxed{\frac{(n-1) \hat{\sigma}^2}{\sigma^2} \sim \chi_{n-1}^2}$$



$$P \left(\chi_{\alpha/2, n-1}^2 \leq \frac{(n-1) \hat{\sigma}^2}{\sigma^2} \leq \chi_{1-\alpha/2, n-1}^2 \right) = 1-\alpha$$

↓ isolate σ^2

$$P \left(\sqrt{\frac{(n-1) \hat{\sigma}^2}{\chi_{1-\alpha/2, n-1}^2}} \leq \sigma \leq \sqrt{\frac{(n-1) \hat{\sigma}^2}{\chi_{\alpha/2, n-1}^2}} \right) = 1-\alpha$$

$$CI = \left[\sqrt{\frac{(n-1) \hat{\sigma}^2}{\chi_{1-\frac{\alpha}{2}, n-1}^2}}, \sqrt{\frac{(n-1) \hat{\sigma}^2}{\chi_{\alpha/2, n-1}^2}} \right]$$