

w9: ~~Monday~~ Wednesday jbradic

Math183Homework6.pdf

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Hypotheses testing (ch 6).

$$X_1, \dots, X_n \sim \text{iid } f_\theta \in \{f_\theta: \theta \in \Theta\}$$

$\hat{\theta}$ ✓.

CI for θ ✓

[For some θ_0]

Is data reasonably compatible with $\theta = \theta_0$?

(not "do we believe $\theta = \theta_0$ exactly")

or is the evidence against $\theta = \theta_0$ strong?

$H_0: \theta = \theta_0$ (null)

$H_1: \theta \neq \theta_0$ (2-sided alternative)

To fix ideas suppose

$$X_1, \dots, X_n \sim N(\mu, \sigma_0^2)$$

unknown known.

test:

$H_0: \mu = \mu_0$, μ_0 - known

vs. $H_1: \mu > \mu_0$ (one-sided alternative)

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Decision $\begin{cases} \rightarrow H_0 \\ \rightarrow H_1 \end{cases}$ but language is delicate

And decision may be wrong either way

We'll either "accept H_0 " in favor of H_1

or

"reject H_0 "

based on ~~some~~ a test-statistic

Test statistic is usually connected to MLE estimator of the parameter we are testing, i.e. of μ .

Suppose the decision rule was
reject H_0 iff $\bar{X} > \mu_0$

Primary fear:

False rejection - type I error

$\mathbb{P}(\bar{X} > \mu_0 \mid \text{when } \mu \text{ is really equal to } \mu_0)$

$$= \mathbb{P}\left(\frac{\bar{X} - \mu_0}{\sigma_0^2/\sqrt{n}} > 0\right) = \underbrace{0.5}_{\text{unacceptably high}}$$

Hence boundary needs margin on the right

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Suppose we are willing to tolerate some false rejection.

level of a test

$$P(\text{false rejection}) = \alpha$$

α - some small number (often 5%, 1%)

$$P(\text{reject} \mid \mu = \mu_0) = P(\bar{X} > \mu_0 + ?) = \alpha.$$

$$\Rightarrow P\left(\frac{\bar{X} - \mu_0}{\sigma_0/\sqrt{n}} > \underbrace{\frac{?}{\sigma_0/\sqrt{n}}}_{z_{1-\alpha}}\right) = \alpha.$$

$\Rightarrow$ Hence Rejection rule is:

Reject iff

$$\bar{X} > \mu_0 + \underbrace{\frac{\sigma_0}{\sqrt{n}} z_{1-\alpha}}_{\text{gets closer to } \mu_0 \text{ with } n \rightarrow \infty}.$$

gets closer to μ_0 with $n \rightarrow \infty$

When $\alpha \rightarrow 0$: harder to reject

$n \rightarrow \infty$: easier to reject

$\sigma_0 \rightarrow \infty$ harder to reject

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Of course, we want to reject when we should. Does this procedure ensures of that?

→ Type II error:

$$\mathbb{P}[\text{type II error}] =$$

$$= \mathbb{P} \left(\bar{X} < \mu_0 + \frac{\sigma_0}{\sqrt{n}} z_{1-\alpha} \mid \mu = \mu_1 \right)$$

↑
point inside H_1

$$= \mathbb{P} \left(\frac{\bar{X} - \mu_0}{\sigma_0/\sqrt{n}} < \frac{\mu_0 - \mu_1 + \frac{\sigma_0}{\sqrt{n}} z_{1-\alpha}}{\sigma_0/\sqrt{n}} \right)$$

$$= \Phi \left(\frac{\mu_0 - \mu_1}{\sigma_0/\sqrt{n}} + z_{1-\alpha} \right)$$

Ideally we want Type II error $\rightarrow 0$