$\boxed{\text{W9: FRIDAY}}$

~~Rejection rule~~

$H_0: \mu = \mu_0$

$H_1: \mu > \mu_0$

$X_1, \dots, X_n \sim N(\underset{\substack{\uparrow \\ \text{unknown}}}{\mu}, \underset{\substack{\uparrow \\ \text{known.}}}{\sigma_0^2})$

Rejection rule:

Reject H_0: iff $\underbrace{\bar{X} > \mu_0 + \frac{\sigma_0}{\sqrt{n}} \cdot z_{1-\alpha}}_{\text{"Rejection region"}}$.

As $\alpha \to 0$ it gets harder to reject

$n \to \infty$ it gets easier to reject

$\sigma_0 \to \infty$ it gets harder to reject

type I error

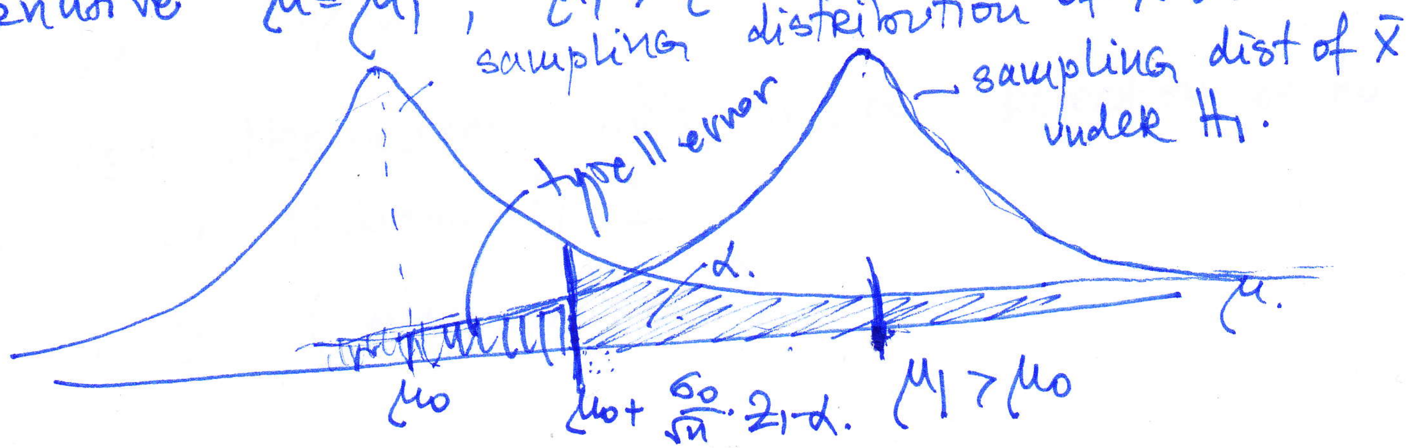
Type II error: $\boxed{\text{we want to reject } H_0 \text{ when we should.}}$

- want to avoid false acceptances.
- we want to accept H_0 when we should.

Does our decision rule ensures this.

$P(\text{accept } H_0 \mid H_0 \text{ is true}) = ? = 1 \checkmark ??$

Assume we are off the null, and on fixed alternative $\mu = \mu_1$, $\mu_1 > \mu_0$.



type II error

def:

$\mathbb{P}$ (accept H_0 | when H_0 is not true).

$= \mathbb{P}$ (accept H_0 | when H_1 is true)

$= \mathbb{P}$ ($\{$reject $H_0\}^c$ | when H_1 is true).

$$= \mathbb{P}\left(\bar{X} < \mu_0 + \frac{\sigma_0}{\sqrt{n}} z_{1-\alpha} \;\middle|\; \underline{\mu = \mu_1,\ \mu_1 > \mu_0} \right).$$

~~under H_0: $\bar{X} \sim \mathcal{N}\left(\mu_0, \frac{\sigma_0}{\sqrt{n}}\right)$.~~

under H_1: $\bar{X} \sim \mathcal{N}\left(\mu_1, \frac{\sigma_0}{\sqrt{n}}\right)$

$$= \mathbb{P}\left(\underbrace{\frac{\bar{X} - \mu_1}{\sigma_0/\sqrt{n}}}_{= Z} < \frac{\mu_0 + \frac{\sigma_0}{\sqrt{n}} z_{1-\alpha} - \mu_1}{\sigma_0/\sqrt{n}} \right).$$

$$= \mathbb{P}\left(Z < \frac{\mu_0 - \mu_1}{\sigma_0/\sqrt{n}} + z_{1-\alpha} \right).$$

$$= \underset{\uparrow \text{ cdf}}{\Phi}\left(\frac{\mu_0 - \mu_1}{\sigma_0/\sqrt{n}} + z_{1-\alpha} \right) \text{ — type II error.}$$

For fixed alternative with $n \to \infty$ rejection of H_0 occurs with probability 1.

$\longrightarrow 0$ (when $n \to \infty$).

Def:

Power is $\mathbb{P}$ (rejecting when you should)

power is 1 − type II error $= 1-\beta$.

depend on a particular poin in the alternative (aka μ_1).

$$\text{power} = 1-\Phi\left(\frac{\mu_0-\mu_1}{\sigma_0/\sqrt{n}} + z_{1-\alpha}\right).$$

$H_0: \mu = \mu_0 = 0.5$

$H_1: \mu > \mu_0$

$\alpha = 5\%$

$H_0: \mu = 0.5 \qquad \sigma_0 = 1, n=100$

$H_1: \mu > 0.5. \qquad \sigma_0 = 1, n=100$

power $\cancel{1-\Phi\left((0.5-\mu_1)\cdot 10\right)}$

$$1-\Phi\left(\frac{0.5-\mu_1}{1/10} + \underbrace{z_{0.95}}_{1.68}\right).$$