HOMEWORK 9: MATH 183 WINTER 2013

DUE <u>IN CLASS</u> ON FRIDAY MARCH 15TH

Reading: Chapters 6.2 & 6.3 & 6.4 of Larsen & Marx.

Reading: 7.4 (third topic) & 7.5 (third topic) of Larsen & Marx.

Explain your answers fully

(1)

   (a) Let $X_1, \cdots, X_{100}$ be a sample drawn from Normal distribution $\mathcal{N}$ with parameters μ and σ_0^2 (for some known value σ_0^2). Derive a statistical hypothesis test for testing $H_0 : \mu \geq 10$ vs $H_1 : \mu < 10$ at 1%- significance level.

   (b) What would change in procedure from part (a) if σ_0^2 was not known. (support your answer with mathematical explanation)

   (c) Would you say that your decision rule from part (a) and part (b) become close enough if the sample size encases to a big number (say 10 000)? (explain your answer the best you can)

   (d) Derive Type II error of your decision rule from part (a) if the true value of μ was 5.

   (e) Derive power of your decision rule from part (a)

(2) Let $X_1, \cdots, X_{100}$ be the following sample drawn from Binomial distribution $\mathcal{B}$ with parameters 100 and p (for some unknown value p). Does the following decision rule

$$\text{reject } H_0 \text{ if and only if } \hat{p}_{MLE} > 0.15 + \sqrt{\frac{0.9 * 0.1}{10}} 1.5$$

for testing $H_0 : p \leq 0.1$ vs $H_1 : p > 0.1$ have good statistical properties? (Explain in full which properties do you expect a good test to have and explain your answer with mathematical derivations)

Solve the following exercises in full from Larsen and Marx textbook.

   (4) 6.4.8.
   (5) 6.4.18.
   (6) 7.4.20.
   (7) 7.5.14.(only part (b) but show all steps in your work)
   (8) 7.5.16.

Challenge problem:

Let $X_1, \cdots, X_n$ and $Y_1, \cdots, Y_n$ be two samples drawn from Normal distributions $\mathcal{N}$ with parameters μ_1 and σ_0^2 and μ_1 and σ_0^2 (for some known value σ_0^2). Derive a statistical hypothesis test for testing $H_0 : \mu_1 - \mu_2 \leq 0$ vs $H_1 : \mu_1 - \mu_2 \geq 0$ at 1%- significance level.