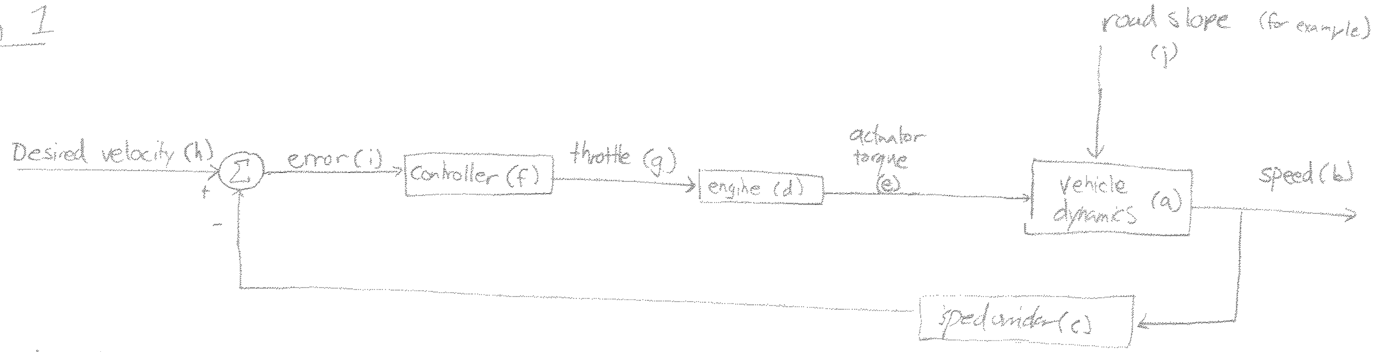
Homework 1 Solutions

units

Problem 1



throttle : flow of fuel/air into engine

torque : twisting effort from the engine

(other disturbances : road slope, frictional losses, etc.)

spedometer : velocity sensor

units:

desired velocity : m/s

error : m/s

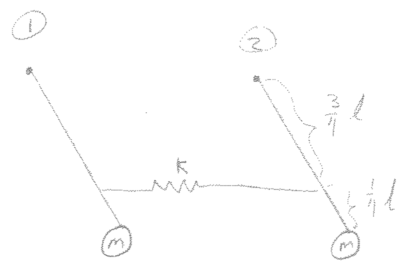
throttle : kg/s

torque : Nm

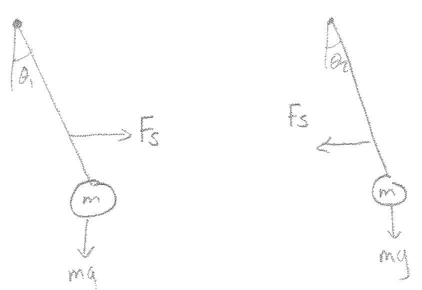
slope : radians

speed : m/s

Problem 2



Draw free body diagrams



Sum the moments

$$\sum M_1 = I_1\ddot{\theta}_1 = -mgl\sin(\theta_1) + F_s\left(\tfrac{3}{4}l\right)\cos\theta_1$$

$$ml^2\ddot{\theta}_1 = -mgl\sin(\theta_1) + K(\Delta x)\left(\tfrac{3}{4}l\right)\cos\theta_1$$

$$ml^2\ddot{\theta}_1 = -mgl\sin(\theta_1) + K\left[\left(\tfrac{3}{4}l\right)(\sin\theta_2 - \sin\theta_1)\right]\left(\tfrac{3}{4}l\right)\cos\theta_1$$

use small angle approximation

$$\sin\theta \approx \theta \qquad \cos\theta \approx 1$$

$$ml^2\ddot{\theta}_1 = -mgl\,\theta_1 + \frac{9}{16}Kl^2(\theta_2 - \theta_1)$$

$$\boxed{0 = ml\ddot{\theta}_1 + mg\theta_1 + \frac{9}{16}Kl(\theta_1 - \theta_2)} \quad \text{or} \quad \boxed{0 = \ddot{\theta}_1 + \frac{g}{l}\theta_1 + \frac{9}{16}\frac{K}{m}(\theta_1 - \theta_2) = 0}$$

Similarly for θ_2

$$\sum M_2 = m l^2 \ddot{\theta}_2 = -mg\, l \sin\theta_2 - F_s \left(\tfrac{3}{4} l\right) \cos\theta_2$$

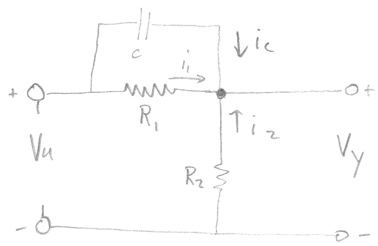
$$m l^2 \ddot{\theta}_2 = -mg\, l \sin\theta_2 - k\left(\tfrac{3}{4} l\right)(\sin\theta_2 - \sin\theta_1)\left(\tfrac{3}{4} l\right)\cos\theta_2$$

$$m l^2 \ddot{\theta}_2 = -mg l\, \theta_2 - \frac{9}{16} k l^2 (\theta_2 - \theta_1)$$

$$\boxed{0 = m l \ddot{\theta}_2 + mg\theta_2 + \frac{9}{16} k l (\theta_2 - \theta_1)} \quad \text{or} \quad \boxed{0 = \ddot{\theta}_2 + \frac{g}{l}\theta_2 + \frac{9}{16}\frac{k}{m}(\theta_2 - \theta_1) = 0}$$

Problem 3

a



sum currents at node

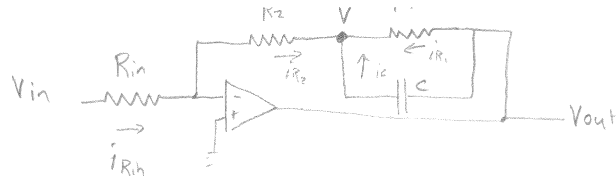
$$i_1 + i_2 + i_c = 0$$

$$\frac{V_u - V_y}{R_1} + C\frac{d}{dt}(V_u - V_y) + \frac{(0 - Y)}{R_2} = 0$$

$$\frac{V_u}{R_1} - \frac{V_y}{R_1} + C(\dot{V}_u - \dot{V}_y) - \frac{V_y}{R_2} = 0$$

$$\boxed{C\dot{V}_u + \frac{1}{R_1} V_u = C\dot{V}_y + \left(\frac{1}{R_1} + \frac{1}{R_2}\right) V_y}$$

c



Sum currents at V

$$i_{R_2} + i_c + i_{R_1} = 0$$

$$(1) \quad \frac{V_{out} - V}{R_1} + \frac{(0 - V)}{R_2} + C(\dot{V}_{out} - \dot{V}) = 0$$

Sum currents going in to op amp

$$i_{R_{in}} = i_{R_2}$$

$$\frac{V_{in} - 0}{R_{in}} = \frac{0 - V}{R_2}$$

$$(2) \quad V = -\frac{R_2}{R_{in}} V_{in}$$

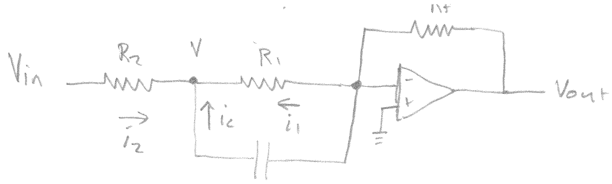
plug (2) into (1)

$$\frac{V_{out} + \frac{R_2}{R_{in}} V_{in}}{R_1} + \frac{\frac{R_2}{R_{in}} V_{in}}{R_2} + C\left(\dot{V}_{out} + \frac{R_2}{R_{in}} \dot{V}_{in}\right) = 0$$

$$\frac{V_{out}}{R_1} + \frac{R_2}{R_1 R_{in}} V_{in} + \frac{1}{R_{in}} V_{in} + C\dot{V}_{out} + C \frac{R_2}{R_{in}} \dot{V}_{in} = 0$$

$$\boxed{\frac{R_2}{R_{in}} C \dot{V}_{in} + \frac{1}{R_{in}}\left(1 + \frac{R_2}{R_1}\right) V_{in} = -C\dot{V}_{out} - \frac{1}{R_1} V_{out}}$$

D



Sum at V

$$i_2 + i_c + i_1 = 0$$

$$\frac{V_{in}-V}{R_2} + C(0-\dot{V}) + \frac{0-V}{R_1} = 0$$

(1) $$\frac{V_{in}-V}{R_2} - C\dot{V} - \frac{V}{R_1} = 0$$

all current flows through R_2 and R_f:

$$i_2 = i_{R_f}$$

$$\frac{V_{in}-V}{R_2} = \frac{0-V_{out}}{R_f}$$

(2) $$V_{in} + \frac{R_2}{R_f} V_{out} = V$$

plug (2) into (1)

$$\frac{V_{in} - \left(V_{in} + \frac{R_2}{R_f} V_{out}\right)}{R_2} - C\left(\dot{V}_{in} + \frac{R_2}{R_f}\dot{V}_{out}\right) - \frac{\left(V_{in} + \frac{R_2}{R_f} V_{out}\right)}{R_1} = 0$$

simplify

$$\boxed{C\dot{V}_{in} + \frac{1}{R_1} V_{in} = \frac{-1}{R_f}\left[R_2 C \dot{V}_{out} + \left(1 + \frac{R_2}{R_1}\right) V_{out}\right]}$$

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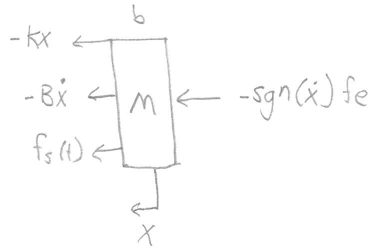
assume $X=0$ when the plates are touching. Additionally, the free length of the spring is where $X=0$

a) draw a free body diagram of b



$sgn(\dot{x})$ returns the sign of $\dot{X}$ (-1 or 1). fe is the electrical resisting force

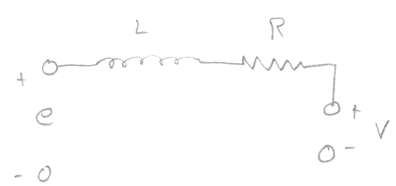
$$\sum F_b = M\ddot{X} = -KX - B\dot{X} + f_s(t) - fe\,sgn(\dot{x})$$

$$M\ddot{X} + B\dot{X} + KX + fe\,sgn(\dot{x}) = f_s(t)$$

$$fe = \frac{q^2}{2\epsilon A}$$

$$(1)\quad M\ddot{X} + B\dot{X} + KX + \frac{q^2}{2\epsilon A}\,sgn(\dot{x}) = f_s(t)$$

draw electrical diagram



Kirchoff's voltage law

$$e = V - IR - L\dot{I}$$

$$I = \dot{q}$$

$$e = V - \dot{q}R - L\ddot{q}$$

e, the voltage on the plate is equivalent to

$$e = \frac{q}{c} = \frac{qx}{\epsilon A}$$

$$\frac{qx}{\epsilon A} = V - R\dot{q} - L\ddot{q}$$

$$\boxed{\begin{gathered} M\ddot{X} + B\dot{X} + KX + \frac{q^2}{2\epsilon A}\,sgn(\dot{x}) = f_s(t) \\ L\ddot{q} + R\dot{q} + \frac{X}{\epsilon A}\,q = V \end{gathered}}$$

b) no, the $sgn(\dot{x})$, q^2, and qx terms prevent a useful linearized system

c) As a microphone, the output is the current I, or equivalently $\dot{q}$

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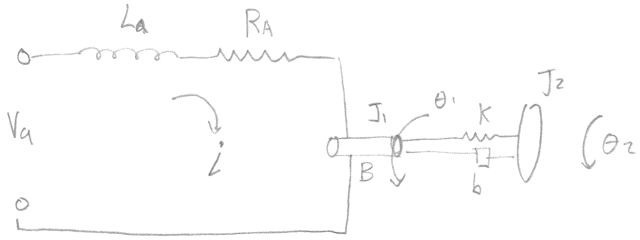

motor torque T_m

Rotor

$$\sum M = J_1\ddot{\theta}_1 = T_m - K(\theta_1-\theta_2) - b(\dot{\theta}_1-\dot{\theta}_2) - B\dot{\theta}_1$$

$$\boxed{J_1\ddot{\theta}_1 = K_t i - K(\theta_1-\theta_2) - b(\dot{\theta}_1-\dot{\theta}_2) - B\dot{\theta}_1}$$

Load

$$\sum M = \boxed{J_2\ddot{\theta}_2 = K(\theta_1-\theta_2) + b(\dot{\theta}_1-\dot{\theta}_2)}$$

circuit

Kirchoff's voltage l:

$$\boxed{V_a - L\frac{d}{dt}i - Ri - K_e\dot{\theta}_1 = 0}$$