Assignment 2: Solving for System Response

ENGR 105: Feedback Control Design
Winter Quarter 2013
Due no later than 4:00 pm on Wednesday, Jan. 23, 2013
Submit in class or in the box outside the door to area of Room 107, Building 550

Problem 1. (5 pts.)

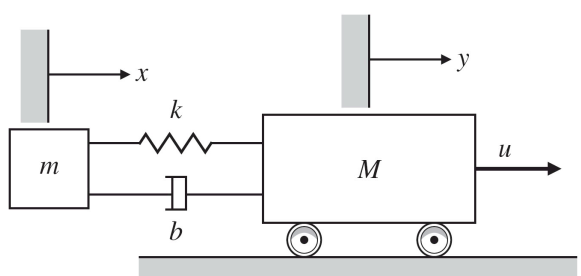
Recall the double-pendulum system for which you found the equations of motion in Assignment 1, Problem 2. Your two variables of interest are θ_1 and θ_2. (Consult the solutions for Assignment 1 to make sure you have the correct equations of motion.)

a. Take the Laplace transform of these equations of motion and solve for $\Theta_1(s)$ and $\Theta_2(s)$ independently.

b. Explain what cool feature of the Laplace transform allows you to do this. And why this is a useful thing to do?

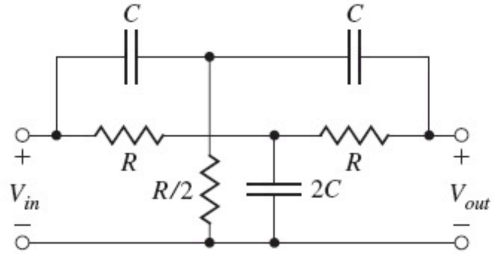
Problem 2. (5 pts.)

In many mechanical positioning systems there is flexibility between one part of the system and another. An example is shown in the figure below, where a force u is applied to the mass M, and another mass m is connected to it. The coupling between the objects is often modeled by a spring constant k with a damping coefficient b, although the actual situation is usually much more complicated than this. Find the transfer function between the control input, u, and the output, y.



Problem 3. (5 pts.)

Find the transfer function for the passive notch circuit shown in the figure below. (This circuit can be used to implement a useful controller using analog components, and we'll examine its use later in the course.)



Problem 4. (10 pts.)

To solve these problems, you are welcome to use a table of Laplace transforms (found in the 6th edition of your book on the inside of the front cover, as well as on page 759).

a. Find the Laplace transform of $f(t) = 1 + 2t$

b. Find the Laplace transform of $f(t) = e^{-t} + 2e^{-2t} + te^{-3t}$

c. Find the time function corresponding to this Laplace transform: $F(s) = \frac{2(s^2+s+1)}{s(s+1)^2}$

d. Solve this ODE using the Laplace transform: $\ddot{y}(t) + \dot{y}(t) + 3y(t) = 0; y(0) = 1, \dot{y}(0) = 2$

e. Solve this ODE using the Laplace transform: $\ddot{y}(t) + y(t) = t; y(0) = 1, \dot{y}(0) = -1$

Problem 5. (10 pts.)

Solve for (1) the impulse response, (2) the unit step response, and (3) the unit ramp response for each of the following transfer functions:

a. $H(s) = \frac{1}{s^2}$

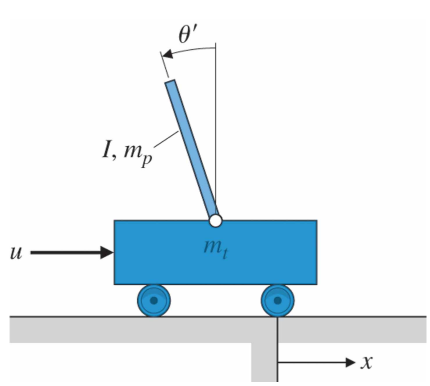
b. $H(s) = \frac{s+1}{s^2}$

c. $H(s) = \frac{s-1}{(s+2)(s+1)}$

d. Which of your resulting responses above are stable?

Problem 6. (15 pts.)

Section 2.1.3 in the 6^{th} edition of the FPE book describes a model for an inverted cart-and-pendulum system that is similar to the Segway scooter. (Note picture of Segway the quad on page 39!) Let's take an advanced look at what is needed to balance an inverted pendulum or Segway.

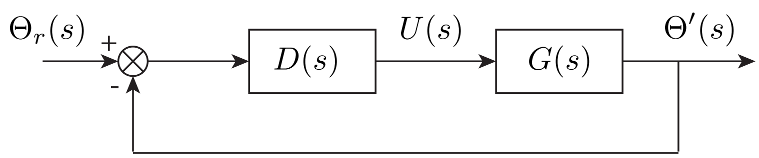


a. First, start with the equations of motion for this system that are given in the book (equation 2.30):

$$\left(I + m_p l^2\right)\ddot{\theta}' - m_p g l \theta' = m_p l \ddot{x}$$
$$(m_t + m_p)\ddot{x} + b\dot{x} - m_p l \ddot{\theta}' = u$$

Derive the transfer function from the cart input $U(s)$ to the angle $\Theta'(s)$. There is a typo in the book (equation 2.31), so your transfer function will be different!

b. Now assume you have a feedback loop closed around the system with a controller $D(s)$ as in the following block diagram:



Derive the closed loop transfer function from reference $\Theta_r(s)$ to pendulum angle $\Theta'(s)$, assuming that the controller is a simple gain, $D(s) = K_p$. You should also assume that $b = 0$ (makes things cleaner for now).

c. Is there a value of K_p that can make the system stable? Why or why not?

d. A slightly more complicated controller has the form $D(s) = K_p + K_d s$. Derive the closed-loop transfer function when this controller is in use.

e. Assuming that $I = m_p = m_t = l = 1$ and that $g = 10$ (instead of 9.81, to make the numbers nice), find K_p and K_d such that both of the closed loop poles are at -1.

f. Using Matlab, plot the unit step response of your closed loop system with these gains to a step change in reference input. Use zero initial conditions. (Submit a printout of your Matlab code and your plot.) What happened? Did you get the response you want?