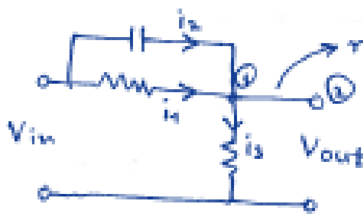
Few points from last week

1) Circuits



no current in this branch
potential of node 1 & 2 is the same
the circuit is not completed there
so no current will flow between 1 & 2
It is just a way of depicting where Vout is being measured.

2) Generally recommended to write equations for θ for rotational systems.

3) Cramer's rule

$$ax + by = e \qquad x = \left(\frac{ed - bf}{ad - bc}\right) \qquad y = \left(\frac{af - ec}{ad - bc}\right)$$
$$cx + dy = f$$

Defn of Laplace transform $\mathcal{L}\{f(t)\} = F(s) = \int_{0^-}^{\infty} f(\tau) e^{-s\tau} d\tau$

Use table to find Laplace transforms of standard functions
can also be used for inverse transforms.

Examples

1) $f(t) = (t+1)^2$

2) $f(t) = t \sin t$

Inverse Laplace - partial fraction

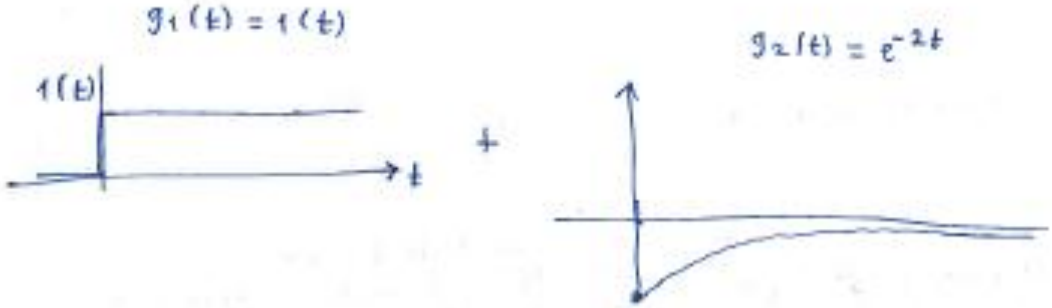
$$F(s) = \frac{2}{s(s+2)} = \frac{c_1}{s} + \frac{c_2}{s+2}$$

$$c_1 = \left(\frac{2}{s+2}\right)\Big|_{s=0} = 1$$

$$c_2 = \frac{2}{s}\Big|_{s=-2} = -1$$

$$F(s) = \frac{1}{s} - \frac{1}{s+2}$$

$$\mathcal{L}^{-1}\{F(s)\} = \mathcal{L}^{-1}\left\{\frac{1}{s}\right\} - \mathcal{L}^{-1}\left\{\frac{1}{s+2}\right\} \Rightarrow f(t) = 1(t) - e^{-2t}\, 1(t)$$



Verify using final value theorem

$$\lim_{t \to \infty} y(t) = \lim_{s \to 0} s\,Y(s)$$

In this case $\lim_{s \to 0} s\,F(s) = \lim_{s \to 0} \cancel{s} \cdot \frac{2}{\cancel{s}(s+2)} = 1$

How to plot this using MATLAB? → (A)

MATLAB code

define a vector for time — `t = [0:0.1:5]`

plot the graph — `plot(t, 1-exp(-2*t))`

HELP command in MATLAB

look at title, xlabel, ylabel, legend commands to make your graph look neater example

controller $\frac{K(s+1)}{(s+2)}$

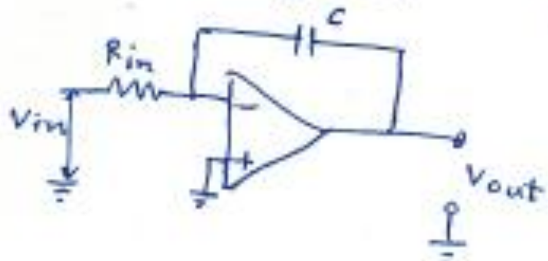
plant $\left(\frac{1}{s+1}\right)\left(\frac{1}{s}\right)$

~~sensor~~ $\cancel{\left(\frac{1}{s+3}\right)}$

construct block diagram and analyze stability wrt K.

concept: open loop poles and closed loop poles

OPAMP Integrator



Finding laplace transform of system
First find the system equation

$$\frac{V_{in}-0}{R_{in}} = C\frac{d(0-V_{out})}{dt}$$

$$\frac{V_{in}(t)}{R_{in}} = -C\frac{dV_{out}(t)}{dt}$$

Take place transform assumption $V_{out}(0)=0$

$$\Rightarrow \quad V_{in}(s) = -R_{in}CsV_{out}(s)$$

$$\Rightarrow \quad \frac{V_{out}(s)}{V_{in}(s)} = \frac{-1}{R_{in}Cs} = G(s) \quad (\text{say})$$

for a step input function $V_{out}(s) = \frac{-1}{R_{in}Cs^2}$

impulse response $V_{out}(t) = \frac{-1}{R_{in}C}1(t)$

step response $V_{out}(t) = \frac{-1}{R_{in}C}\left(\frac{t}{2}\right) \longrightarrow$ ramp

Integrator $\rightarrow$ keeps accumulati

Q) What

④

<u>example</u>

solving linear ODEs

$$y'' - 5y' + 6y = 0 \quad y(0) = 2 \quad y'(0) = 2$$

$$s^2 Y(s) - sy(0) - y'(0) - 5sY(s) + 5y(0) + 6Y(s) = 0$$

$$\Rightarrow Y(s) = \left(\frac{2s-8}{s^2-5s+6}\right) = \frac{(2s-8)}{(s-2)(s-3)}$$

partial fractions

$$Y(s) = \frac{4}{s-2} + \frac{-2}{s-3}$$

$$\Rightarrow y(t) = 4e^{2t} - 2e^{3t}$$

---

partial fractions example

$$Y(s) = \frac{10}{(s-1)(s^2+4s+5)} = \frac{1}{(s+2)^2+1} + \left(\frac{1}{s-1}\right)$$

$$y(t) = e^{2t}\sin t + e^{t}$$