ENGR 105: Feedback Control Design
Winter 2013

Lecture 9 - Performance Specifications

Monday, January 28, 2013

Today's Objectives

1. plot the step response of a second-order system
2. define several performance criteria

Reading: FPE Sections 3.3 & 3.4

1 Step response

Usually when putting performance specifications on the system, we look at the transfer function from the reference to the output:

The most common input used to specify performance is the *unit step*. In most cases, it is much easier in practice to produce something resembling a step change in the reference than it is to produce an impulse.

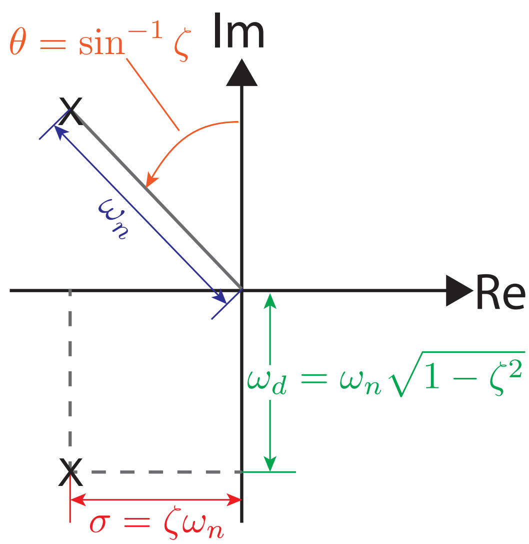
For the car example, we have:

Does a step change in $r(t)$ make any sense? This would be a step change in the position of the lead car which is not physically possible. However, a step change in velocity is more reasonable:

So we can think of the step response as also describing the change in speed of the following vehicle if the lead vehicle speed suddenly changes.

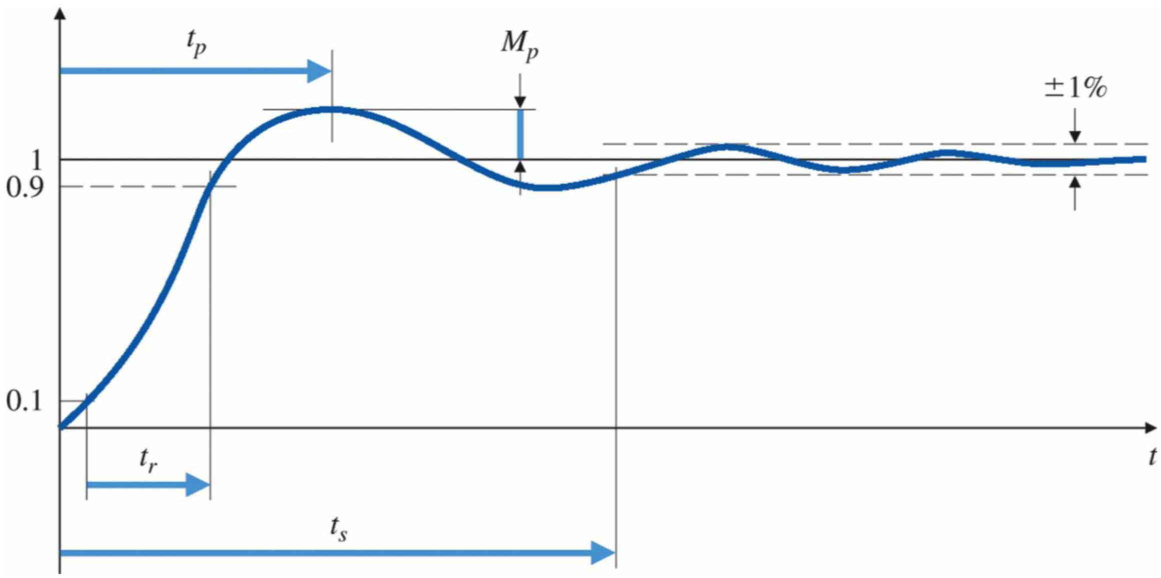
This transfer function has a particular form that makes it easy to specify the step response. More complicated systems will often resemble this response even if they don't fit it exactly, if they have a dominant set of closed-loop poles (a single pair close to the imaginary axis).

We can consider the response in terms of a natural frequency, ω_n, and a damping ratio, ζ.



For a step response we have:

In general this will yield the following type of response:



See Section 3.3 for more examples with different damping ratios

2 Performance criteria

For the response above, we can create a number of performance specifications:

Rise time

Time until the response reaches the vicinity of its new steady state value (approximately 90%). This is based on observation and is not exact.

Peak time

Time until the response (if it is stable) reaches its maximum value. This occurs at the top of the peak which corresponds to a zero slope. We can derive an exact solution for the peak time, t_p.

Peak overshoot

This is maximum overshoot of the system, which occurs at $t = t_p$.

If $\quad \zeta = 0.5, \quad M_p = 0.16$
$\quad \zeta = 0.7, \quad M_p = 0.05$

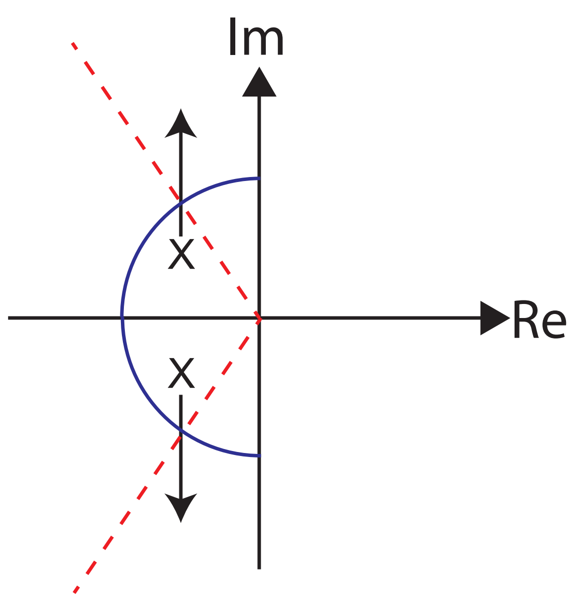
Settling time (1%)

The time at which the response enters a band that is within 1% of the final value. Settling times can be similarly defined at other percentages of the final value.

When $e^{-\sigma t} = 0.01$, the response must be within 1% of the final value of 1

These have nice graphical representation in the s-plane:

Returning to our car-following example above, what can we do by changing the proportional gain?



We can make a damping spec
with a long rise time
-or-
we can make a rise time spec
with low damping.

The gain does not affect the
settling time.