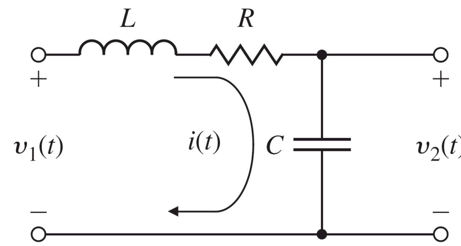
Assignment 4: Performance Specifications and Error Response

ENGR 105: Feedback Control Design
Winter Quarter 2013
Due no later than 4:00 pm on Wednesday, Feb. 6, 2013
Submit in class or in the box outside the door to area of Room 107, Building 550

Problem 1. (10 pts.)



For the electrical circuit shown in the figure above:

a. Find the time-domain equation relating $i(t)$ and $v_1(t)$.

b. Find the time-domain equation relating $i(t)$ and $v_2(t)$.

c. Find the transfer function $\frac{V_2(s)}{V_1(s)}$.

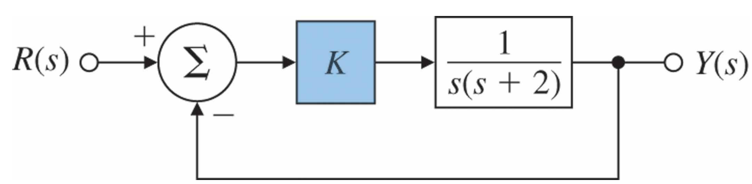
d. Find the damping ratio, ζ, and undamped natural frequency, ω_n, of the system.

e. Find the values of R that will result in $v_2(t)$ having an overshoot of no more than 25%, assuming $v_1(t)$ is a unit step, $L = 10$ mH, and $C = 4$ μF.

f. Plot the response of $v_2(t)$ for the same scenario as in part e, but use the value of R that exactly results in an overshoot of 25%. Annotate the plot to indicate the time at which the maximum overshoot occurs (known as the peak time, t_p). Check that the t_p you see in your plot is a reasonable approximation of the t_p you calculate from ζ and ω_n. Submit your code and annotated plot.

Problem 2. (10 pts.)

a. For the unity gain feedback system shown below, find the gain K of the proportional controller so the output $y(t)$ has an overshoot of 10% in response to a unit step.



b. Does increasing K increase or decrease the overshoot?

c. Provide a Matlab simulation (code and plot printout) that shows the unit step response of the system for the value of the gain K you found in part a, as well as ½ of that value of K. Your plot should show both responses on a single graph (use the `hold on` command) clearly distinguishable and labeled (annotated or using the `legend` command).

d. Discuss in a few sentences the possible advantages and disadvantages of the system behavior for the cases of K and ½ K from part b.

Problem 3. (10 pts.)

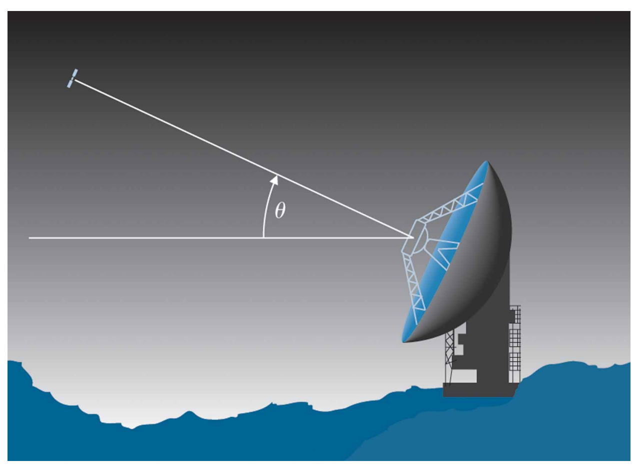
Consider a plant with second-order dynamics, whose behavior is captured by a natural frequency, ω_n, and damping ratio, ζ. Now we add unity feedback with a proportional controller gain, K, which results in a stable closed-loop second-order system. We desire for the system response to a step input to have a peak time of less than t_1.

a. In a sketch of the s-plane (the real-imaginary plane), shade the region that corresponds to the possible locations of poles that meet this criterion. Show all your work, and label your sketch with known points based on the system parameters given above.

b. Determine the allowable values of K for a peak time of less than t_1 as a function of the system parameters.

Problem 4. (10 pts.)

You wish to control the elevation of the satellite-tracking antenna shown in the figures below.



The antenna and drive parts have a moment of inertia J and a damping B; these arise to some extent from bearing an aerodynamic friction, but mostly from the back emf of the DC drive motor. The equation of motion is

$$J\ddot{\theta} + B\dot{\theta} = T_c,$$

where T_c is the torque from the drive motor. Assume that $J = 600{,}000$ kg•m^2 and $B = 20{,}000$ N•m•sec.

a. Find the transfer function between the applied torque, T_c, and the antenna angle, θ.

b. Suppose the applied torque is computed so that θ tracks a reference command, θ_r, according to the feedback law

$$T_c = K(\theta_r - \theta),$$

where K is the feedback gain. Find the transfer function between θ_r and θ.

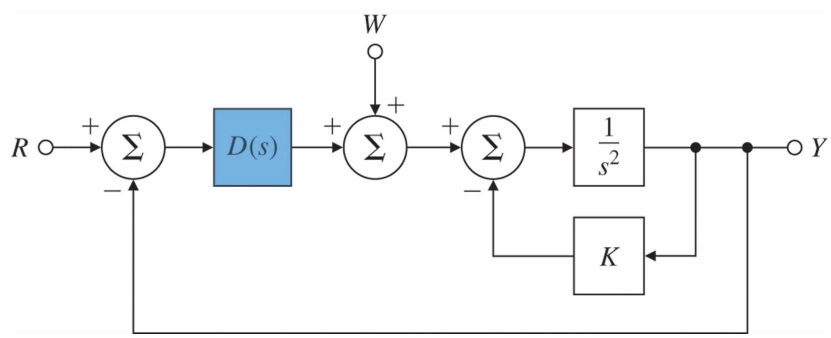
c. What is the maximum value of K that can be used if you wish to have an overshoot $M_p < 10\%$?

d. What values of K will provide a rise time of less than 80 sec? (Ignore the M_p constraint from part c.)

e. Use Matlab to plot the step response of the antenna system for K values of 200, 400, 1000, and 2000. Find the overshoot and rise time of the four step responses by examining your plots. Do the plots confirm your calculations in parts c and d?

Problem 5. (10 pts.)

Consider the system shown in the figure below, which represents control of the angle of a pendulum that has no damping.



a. What condition must $D(s)$ satisfy so that the system can track a ramp reference input with constant steady-state error?

b. For a transfer function $D(s)$ that stabilizes the system and satisfies the condition in part a, find the class of disturbances $w(t)$ that the system can reject with zero steady-state error.