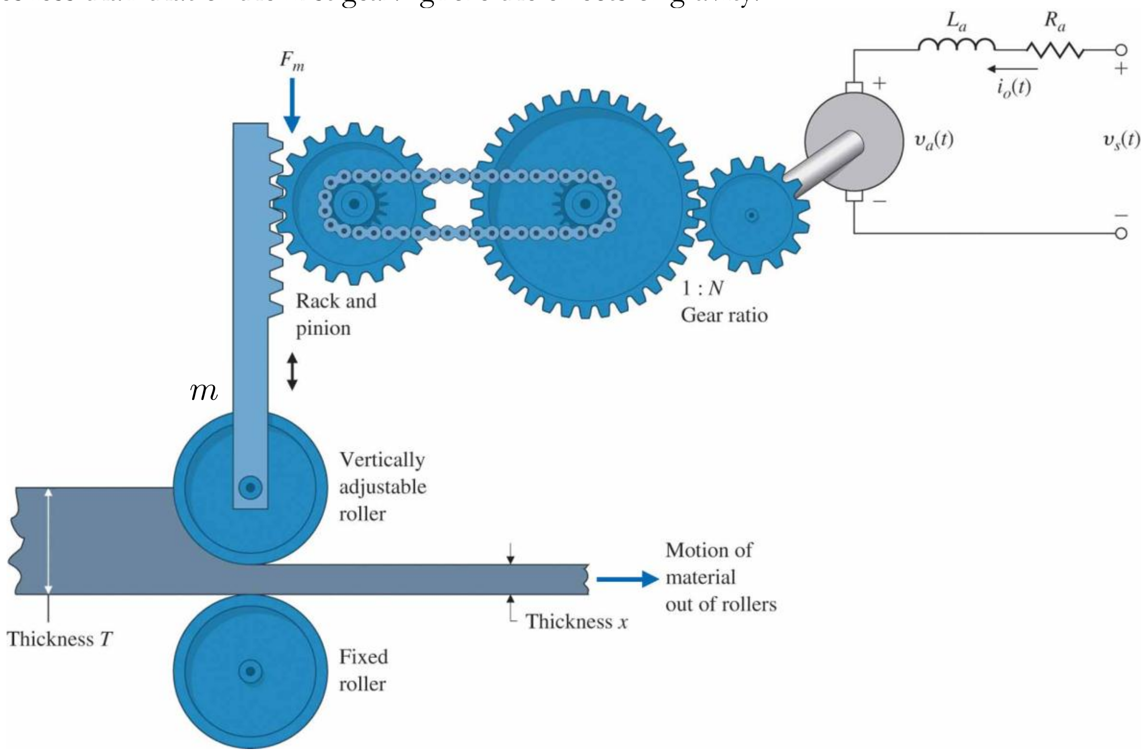
Assignment 3: Block Diagrams and Response to Sinusoidal Inputs

ENGR 105: Feedback Control Design
Winter Quarter 2013
Due no later than 4:00 pm on Wednesday, Jan. 30, 2013
Submit in class or in the box outside the door to area of Room 107, Building 550

Problem 1. (12 pts.)

Consider the continuous rolling mill depicted in the figure below. Suppose that the motion of the adjustable roller has a damping coefficient b, and that the force exerted by the rolled material on the adjustable roller is proportional to the material's change in thickness: $F_s = c(T - x)$. Suppose further that the DC motor has a torque constant K_t and a back EMF constant K_e, and that the rack-and-pinion has effective radius R. Assume that the vertically adjustable roller has mass m. The gears have a combined gear ratio of N; this means that the torque that the last gear (the "rack and pinion") exerts on the roller is N times greater than the torque on the motor. Also, the rotational velocity of the last gear is N times less than that of the first gear. Ignore the effects of gravity.



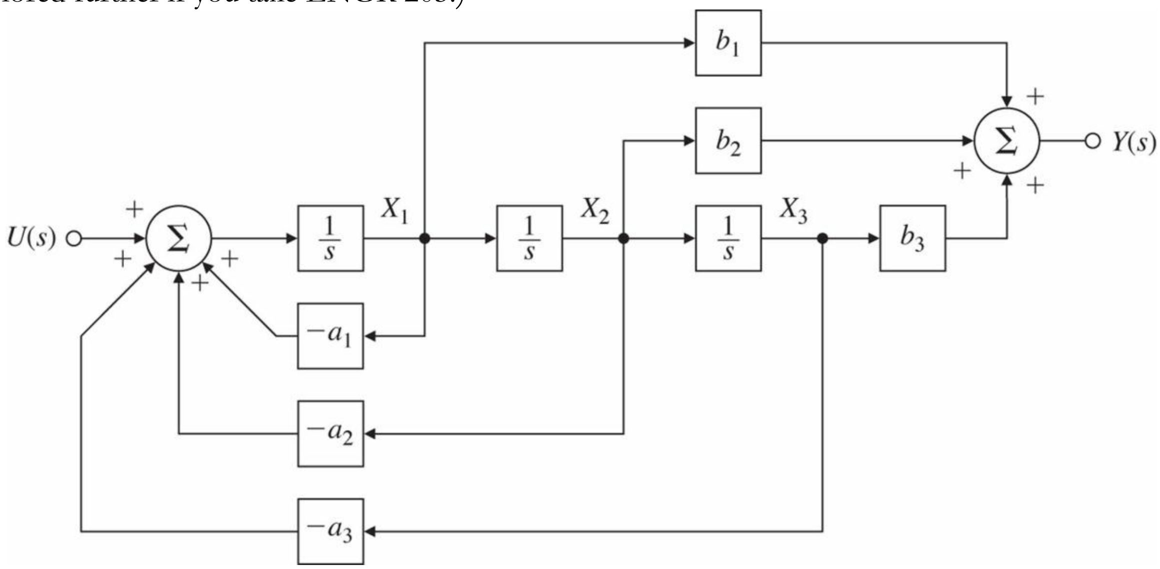
a. What is the controlled input to the system? What is the output? What is the disturbance?

b. Draw a block diagram of the system that explicitly shows the following quantities: $V_s(s)$, $I_o(s)$, $F(s)$ (the force the motor exerts on the adjustable roller), and $X(s)$ (the thickness of the material after rolling). Hint: The current through the circuit obviously affects the motion of the roller (a "feedforward" effect), but note that the movement of the roller in turn affects the dynamics of the circuit through the back-emf constant (a "feedback" effect). Your block diagram should reflect this intuition.

c. Simplify your block diagram as much as possible while still identifying the input, output, and disturbance separately.

Problem 2. (8 pts.)

Consider the block diagram shown below. Note that a_i and b_i are constants. Compute the transfer function for this system. (This special structure is called the "control canonical form" and will be explored further if you take ENGR 205.)



Problem 3. (10 pts.)

Consider the following second-order system with an extra pole:

$$H(s) = \frac{\omega_n^2 p}{(s+p)(s^2 + 2\zeta\omega_n s + \omega_n^2)}$$

a. The Laplace Transform of the step response, $Y(s)$, can be written (using partial fraction expansion) as:

$$Y(s) = \frac{\alpha_1}{s} + \frac{\alpha_2}{s+p} + \frac{\alpha_3}{s - (-\sigma + j\omega_d)} + \frac{\bar{\alpha}_3}{s - (-\sigma - j\omega_d)}$$

where $\bar{\alpha}_3$ is the complex conjugate of α_3. Find α_1, α_2, σ, and ω_d in terms of ω_n, ζ, and p.

b. Show that the unit-step response can be expressed as:

$$y(t) = 1(t) + Ae^{-pt} + Be^{-\sigma t}\sin(\omega_d t - \theta)$$

and find the constants A, B, and θ, in terms of α_1, α_2, $|\alpha_3|$, and $\angle\alpha_3$, (the magnitude and angle of the complex variable α_3.) *Hint: It may be helpful to show that*

$$\mathrm{Re}(\alpha)\cos(\omega t) + \mathrm{Im}(\alpha)\sin(\omega t) = |\alpha|\cos(\omega t - \angle\alpha)$$

where $\mathrm{Re}(\alpha)$ *and* $\mathrm{Im}(\alpha)$ *denote the real and imaginary parts of* α*, respectively.*

c. Using your results for parts a and b, and assume:

$$|\alpha_3| = \frac{p}{2\sqrt{(p^2-2\zeta\omega_n p+\omega_n^2)(1-\zeta^2)}} \qquad \angle\alpha = \tan^{-1}\frac{\sqrt{1-\zeta^2}}{-\zeta} + \tan^{-1}\frac{\omega_n\sqrt{1-\zeta^2}}{p-\zeta\omega_n} + \frac{\pi}{2}$$

Which term dominates $y(t)$ as p gets large?

d. Give approximate values of A and B for small values of p.

e. Which term dominates $y(t)$ as p gets small? (State what it is getting small with respect to.)

f. Using the given explicit expression for $y(t)$ or the step command in Matlab, and assuming $\omega_n = 1$ and $\zeta = 0.7$, use MATLAB to plot the step response of the system for several values of p ranging from very small to very large. At what point does the extra pole cease to have much effect on the system response?

Problem 4. (10 pts.)

Determine the final value to a step input (make sure to include the step) for each of the following transfer functions:

a. $G(s) = \frac{s+2}{s+1}$

b. $G(s) = \frac{s+2}{s(s+1)}$ (Careful! Describe the reason for any issue you might encounter with this transfer function responding to a step input.)

c. $G(s) = e^{-Ts}$ (This is a pure time delay, where T is the amount of the delay.)

d. $G(s) = \frac{1-{Ts}/{2}}{1+{Ts}/{2}}$ (This is known as the first-order Padé approximation of time delay. How does the final value to a step input correspond to that of the exact time delay given in part c?)

Problem 5. (10 pts.)

A notch filter is a way of removing certain frequencies in a system. Show that if the transfer function of a notch filter is described by

$$G(s) = \frac{s^2+\omega^2}{s^2+2\zeta\omega s+\omega^2} \; ,$$

for some positive values of ω and ζ, the filter output will decay to zero when the input is a sinusoid of frequency ω.